ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 2, Issue 12, December 2017

Anti-Fuzzy ideals in Boolean like Semi – rings

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Abstract:-- **In this paper we introduce the notion of anti-fuzzy ideals in Boolean like semi – ring R and also obtain some of their properties. Let R be a Boolean like semi – ring and let μ be a fuzzy set defined on R. Then μ is said to be a anti fuzzy ideal of R if** $i)\mu(x-y) \leq \max\{\mu(x), \mu(y)\}$ **ii)**$\mu(ra) \leq \mu(a)$ $iii)\mu((r+a)s+rs) \leq \mu(a)$ $for\ all\ r, a, s \in R.$

Keywords:-- **Boolean like semi ring, fuzzy set , fuzzy ideal, Anti fuzzy ideal.**

1. INTRODUCTION

Boolean like semi rings were introduced in role by K.Venkatesawarlu, B.V.N. Murthy and N. Amaranth[6] during 2011.Boolean like rings of A.L. Foster arise naturally from general ring dulity considerations and preserve many of the formal properties of Boolean ring. A Boolean like ring is a commutative ring with unity and is of characteristic 2. It is clear that every Boolean ring is a Boolean like ring but not conversely. The concept of a fuzzy subset of a nonempty set was introduced by Zadeh[7]. Fuzzy ideals of rings were introduced by Ziu, and it has been studied by several authors. The notion of fuzzy ideals and its properties were applied to various areas: Semi groups, Bck-algebras and semi rings.R.Biswas [1]introduced the concept of anti fuzzy subgroups and K.H.Kim and Y.B.Jun[4] studied the notion of anti fuzzy ideals in near rings. In this paper we introduce the concept of anti fuzzy ideals in Boolean like semi rings and study the some properties of anti fuzzy ideals.

§.2 PRELIMINARIES

Definition 2.1
A non empty set R with two binary operations ‘+’ and ‘·’ is called a **near – ring** if

i) (R,+) is a group (not necessarily abelian)
ii) (R,·) is a Semigroup
iii) x.(y+z) =x.y+x.z for all x,y,z∈R

Definition 2.2
A system(R,+,·) a **Boolean semi ring** iff the following properties hold

i) (R,+) is an additive (abelian) group(whose ‘zero’ will be denoted by ‘0’)
ii) (R,·) is a semigroup of idempotents in the sense aa=a, for all a∈R.
iii) a(b+c)=ab+ac and
iv) abc=bac,for all a,b,c $\in R$

Example 2.3
Let (G, +) be any abelian group define ab = b for all a,b∈G. Then (G,+, ·) is a Boolean Semiring.

Definition 2.4
A nonempty set R together with two binary operations + and · satisfying the following conditions is called a **Boolean like semi ring**.

i) (R,+) is an abelian group
ii) (R,.) is a semi group
iii) a. (b + c) = a. b + a.c for all a,b,c $\in R$
iv) a + a = 0 for all a in R
v) ab (a + b + ab) = ab for all a,b $\in R$

Definition 2.5
A nonempty subset I of R is said to be an **ideal** if

i) (I,+) is a subgroup of (R,+),(ie)., for a,b ∈ R => a+b ∈ R.
ii) ra ∈ R for all a ∈ I, r ∈ R(ie).,RI ⊆ I
iii) (r+a)s+rs ∈ I for all r,s ∈ R, a ∈ I.

Definition 2.6
Let μ be a fuzzy set defined on R. Then μ is said to be a **fuzzy ideal** of R if

i) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}, x, y \in R$
ii) $\mu(ra) \geq \mu(a)$ for all $r, a \in R$
iii) $\mu((r+a)s+rs) \geq \mu(a)$ for all $r, a, s \in R$

§.3 MAIN RESULTS

Definition 3.1

A Fuzzy set μ in a Boolean like semi ring **R** is called an anti-fuzzy left ideal of M, if

i) $\mu(x-y) \leq max\{\mu(x),\mu(y)\}, x,y \in \boldsymbol{R}$

ii) $\mu(ra) \leq \mu(a), \forall\, r,a \in \boldsymbol{R}$

iii) $\mu((r+a)s+rs) \leq \mu(a), \forall\, r,a,s \in \boldsymbol{R}$

Example 3.2

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	a	b	a	0

·	0	a	b	c
0	0	0	0	0
a	0	0	a	A
b	0	0	b	b
c	0	a	b	c

Clearly it is a Boolean – like – semi ring. Let μ be an Anti-fuzzy ideal defined on **R** by μ(x) = 0.6 for every $x \in M$. Then μ is an anti-fuzzy ideal of M.

Theorem 3.3

Let **R** be a Boolean like semi ring and μ be an anti-fuzzy left (respectively right) ideal of **R.** Then the set $\mathbf{R}_\mu = \{ x \in \boldsymbol{R} \,/\, \mu(x) = \mu(0) \}$ is a left (respectively right) ideal of **R.**

Proof

Let μ be an anti-fuzzy left ideal

i) Let $x, y \in R\mu$ implies μ(x) = μ(0) and μ(y) = μ(0). Then μ(x-y) ≤ max { μ(x),μ(y) } which implies that μ(x-y) ≤ max { μ(0),μ(0) } = μ(0). Hence $x - y \in R_\mu$.

ii) Now for every $r, s \in R, a \in R\mu$. $\mu((r+a)s + rs) \leq \mu(a) = \mu(0) \Rightarrow \mu((r+a)s+rs) = \mu(0)$ [respectively $\mu(ra) \leq \mu(a) = \mu(0) \Rightarrow \mu(ra) = \mu(0)$]

(ie) $(r+a)s + rs \in R\mu$

[respectively $ra \in R\mu$].

Theorem 3.4

If $\{ \mu i \,/\, i \in \Lambda \}$ is a family of anti fuzzy ideals of a Boolean like semi ring **R** then so is $\vee_{i \in I}\, \mu_i$

Proof

Let $\{ \mu i \,/\, i \in \Lambda \}$ be a family of anti fuzzy ideals of **R** and let $x, y \in \boldsymbol{R}$. Then

$(\vee_{i\in I}\, \mu_i)(x-y) = \sup\{ \mu_i(x-y) / i \in \Lambda \} \leq \sup\{ \max\{ \mu_i(x), \mu_i(y) / i \in \Lambda \}\} = \max\{ \sup\{ \mu_i(x) / i \in \Lambda\}, \sup\{ \mu_i(y) / i \in \Lambda \}\}$

$= \max\{ (\vee_{i\in I}\, \mu_i)(x), (\vee_{i\in I}\, \mu_i)(y) \}$

and let $r, a \in \boldsymbol{R}$. Then,

$(\vee_{i\in I}\, \mu_i)(ra) = \sup\{ \mu_i(ra) / i \in \Lambda \}$

$\leq \sup\{ \mu_i(a) / i \in \Lambda \}$

$= (\vee_{i\in I}\, \mu_i)(a)$

Now, let $r, a, s \in \boldsymbol{R}$. Then,

$(\vee_{i\in I}\, \mu_i)((r+a)s + rs) = \sup\{ \mu_i((r+a)s + rs) / i \in \Lambda \} \leq \sup\{ \mu_i(a) / i \in \Lambda \} = (\vee_{i\in I}\, \mu_i)(a)$

Theorem 3.5

Intersection of a non-empty collection of anti fuzzy left (resp.right) ideals of a Boolean like semi ring **R** is an anti fuzzy left (resp.right) ideal of **R**.

Proof

Let **R** be a Boolean like semi ring. Let $\{ \mu_i / i \in I \}$ be the family of anti fuzzy left(resp.right) ideal of **R** and let $x, y \in \boldsymbol{R}$. Then, we have

i) $(\cap_{i \in I}\, \mu_i)(x-y)$

$= \inf_{i\in I}\{ \mu_i(x-y) \}$

$\leq \inf_{i\in I}\{ \max\{ \mu_i(x), \mu_i(y) \}\}$

$= \max\{ \inf_{i\in I}\mu_i(x), \inf_{i\in I}\mu_i(y) \}$

$= \max[(\cap_{i \in I}\, \mu_i)(x), (\cap_{i \in I}\, \mu_i)(y)]$

ii) Let $r, a \in \boldsymbol{R}$. Then,

$(\cap_{i \in I}\, \mu_i)(ra) = \inf_{i\in I}\{ \mu_i(ra) \}$

$\leq \inf_{i\in I}\{ \mu_i(a) \}$

$= (\cap_{i \in I}\, \mu_i)(a)$

iii) Let $r, a, s \in \boldsymbol{R}$. Then,

International Journal of Science, Engineering and Management (IJSEM)
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$(\bigcap_{i \in I} \mu_i)((r + a)s + rs)$

$= \inf_{i\in I}\{ \mu_i((r + a)s + rs) \}$

$\le \inf_{i\in I}\{ \mu_i(a) \} = (\bigcap_{i \in I} \mu_i)(a)$

Theorem 3.6

Let **R** be a Boolean like semi ring. Then a fuzzy set μ is an anti fuzzy ideal of **R** iff μ^c is a fuzzy ideal of **R**.

Proof

Let $x, y \in R$ and μ be an anti fuzzy ideal of **R** then we have,

i) $\mu^c(x - y) = 1 - \mu(x - y)$

$\ge 1 - \max\{ \mu(x), \mu(y)\}$

$= \min\{ 1 - \mu(x), 1 - \mu(y) \}$

$= \min\{ \mu^c(x), \mu^c(y) \}$

ii) Let $r, a \in R$. Then,

$\mu^c(ra) = 1 - \mu(ra)$

$\ge 1 - \mu(a) = \mu^c(a)$

iii) Let $r, a, s \in R$. Then,

$\mu^c((r + a)s + rs)$

$= 1 - \mu((r + a)s + rs) \ge 1 - \mu(a)$

$= \mu^c(a)$

Hence μ^c is a fuzzy ideal of **R** similarly the converse follows.

Theorem 3.7

A Boolean like semi ring homomorphic pre-image of an anti fuzzy ideal is an anti fuzzy ideal.

Proof

Let R and S be Boolean like semi rings. Let f : R → S be a Boolean like semi ring homomorphism ϑ be an anti fuzzy ideal of S and μ be the pre image of ϑ under f. Let $x, y, r, a, s \in R$. Then,

i) $\mu(x - y) = \vartheta(f(x - y))$

$= \vartheta(f(x) - f(y))$

$\le \max\{\vartheta(f(x)), \vartheta(f(y))\}$

$= \max\{\mu(x), \mu(y)\}$

ii) $\mu(ra) = \vartheta(f(ra)) = \vartheta(f(r)f(a)) \le \vartheta(f(a)) = \mu(a)$

iii) $\mu((r + a)s + rs) = \vartheta(f(r + a)s + rs)$

$= \vartheta(f((r + a)s) + f(rs))$

$= \vartheta(f(r + a).f(s) + f(rs))$

$= \vartheta((f(r) + f(a))f(s)) + f(r)f(s))$

$= \vartheta(f(r)f(s) + f(a)f(s)f(r)f(s))$

$= \vartheta(f(a)) = \mu(a)$

Hence μ is an anti fuzzy ideal of **R.**

Theorem 3.8

Let μ be an anti fuzzy left (resp.right) ideal of a Boolean like semi ring **R** and μ^+ μ^+ be a fuzzy set in **R** given by $\mu^+(x) = \mu(x) + 1 - \mu(1)$ for all $x \in R$. Then μ^+ is an anti fuzzy left (resp.right) ideal of **R**.

Proof

Let μ be an anti fuzzy left ideal of a Boolean like semi ring **R** for all $x, y, r, a, s \in R$. Then,

i) $\mu^+(x - y) = \mu(x - y) + 1 - \mu(1)$

$\le \max\{ \mu(x), \mu(y) \} + 1 - \mu(1)\} = \max\{ \mu(x) + 1 - \mu(1), \mu(y) + 1 - \mu(1)\} = \max\{ \mu^+(x), \mu^+(y) \}$

ii) $\mu^+(ra) = \mu(ra) + 1 - \mu(1)$

$\le \mu(a) + 1 - \mu(1)$

$= \mu^+(a)$

iii) $\mu^+((r + a)s + rs) = \mu((r + a)s + rs) + 1 - \mu(1)$

$\le \mu(a) + 1 - \mu(1) = \mu^+(a)$

Hence μ^+ is an anti fuzzy left ideal of a Boolean like semi ring **R.**

Theorem 3.9

Let **R** be a Boolean like semi ring. Then a fuzzy set μ is normal anti fuzzy left (resp.right) ideal of Boolean like semi ring **R** iff $\mu^+ = \mu$

Proof

Sufficient is directly follows.

ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 2, Issue 12, December 2017

T.P : The Necessary Part

Suppose μ is normal anti fuzzy left (res.right) ideal of **R.**

Then

$\mu^{+}(x) = \mu(x) + 1 - \mu(1) = \mu(x) + 1 – 1 = \mu(x)$ for all

$x \in \boldsymbol{R}$. Hence , $\mu^{+} = \mu$

Theorem 3.10

Let μ be an anti fuzzy left (res.right) ideal of a Boolean like semi ring **R** then $(\mu^{+})^{+} = \mu^{+}$

Proof

For any $x \in \boldsymbol{R}$,we have $(\mu^{+})^{+}(x) = \mu^{+}(x) + 1 - \mu(1) = \mu(x) + 1 - \mu(1) = \mu^{+}(x)$.

Hence $(\mu^{+})^{+} = \mu^{+}$.

Theorem 3.11 Let μ be an anti fuzzy left (resp.right) ideal of a Boolean like semi ring $\boldsymbol{R}$ & ϕ: $[0, \mu(0)]\to[0,1]$ be an increasing function. Let μ_{ϕ} be a fuzzy set in **R** defined by $\mu_{\phi}(x) = \phi\left(\mu(x)\right)$, for all $x \in \boldsymbol{R}$. Then μ_{ϕ} is an anti fuzzy left (resp.right) ideal of **R**.

Proof

Let $x, y, r, a, s \in \boldsymbol{R}$. Then

i) $\mu_{\phi}(x – y) = \phi(\mu(x - y))$

$\leq \phi\,(\,max\,\{\,\mu(x)\,,\mu(y)\,\}$

$= \max\{\phi\left(\mu(x)\right), \phi\left(\mu(y)\right)\} = max\,\{\,\mu_{\phi}(x)\,,\mu_{\phi}(y)\,\}$

ii) $\mu_{\phi}(ra) = \phi\,(\mu(ra)) \leq \phi\,(\mu(a)) = \mu_{\phi}(a)$

iii) $\mu_{\phi}\left((r + a)s + rs\right) = \phi\,\mu\left((r + a)s + rs\right) \leq \phi\left(\mu(a)\right) = \mu\phi(a)$.

Hence μ_{ϕ} is an anti fuzzy left ideal of **R.**

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