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Ascending Tree Cover of Some Special Graphs

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***Abstract:*-- Ascending tree cover of a graph G is a partition of edges of G into trees $G_1, G_2, _3, \ldots\ldots\ldots G_n$ such that $|E(G_i)| = i$ for all i = 1 to n. In this paper, we investigate the ascending tree cover for P_n^+ and C_n^+.**

***Keywords:*-- Continuous Monotonic Decomposition, Ascending cover, Ascending tree cover.**
AMS Subject Classification: 05C70

1. INTRODUCTION

By a graph, we mean a finite, undirected, non-trivial, connected graph without loops and multiple edges. In [4], GnanaDhaset. al. introduced the concept of Continuous Monotonic Decomposition of Graphs. A decomposition (G_1, G_2, G_3 ,…,G_n) of G is said to be Continuous Monotonic Decomposition (CMD) if each $G_i$connected and $|E(G_i)|$ = i for each i = 1,2…n. If each G_i is isomorphic to a tree, it is known as Continuous Monotonic Tree Decomposition of G or Ascending Tree cover of G.

Definition 1.1:

The Corona$G_1 \odot G_2$of two graphs $G_1$and $G_2$is defined as the graph obtained by taking one copy of G_1(with $p_1$vertices) and $p_1$copies of $G_2$and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2. In particular, the graph $G \odot K_1$ is denoted by G^+. The graph P_n^+ is called Comb.

Definitions which are not seen here can be found in [1] and [2]. In this paper, we initiate a study on Ascending Tree Cover for$P_n^+$and C_n^+.

2. MAIN RESULTS

Definition 2.1:

Ascending Tree Cover (ATC) of G is defined as a decomposition of G into edge-disjoint sub graphs $G_1, G_2, \ldots, G_n$ such that

(i) Each sub graph is isomorphic to a tree.

(ii) $|E(G_i)|$ = i for each i = 1,2…n.

Lemma 2.2:
If a (p,q) graph G admits ATC into n parts then p ≥ n+1.

Proof:

As n = $|E(G_n)| \leq$ p-1 , we have p ≥ n+1.

Theorem 2.3:

P_n^+, n ≥ 2 admits ATC into q parts iff q≡1 or 2 (mod 4) and 4n = q^2 + q + 2.

Proof:

Let $V(P_n^+)=\{u_1,u_2,\ldots,u_n,v_1,v_2,\ldots,v_n\}$

$E(P_n^+)=\{(u_i,u_{i+1})|1\leq i\leq n-1\} \cup \{(u_1,u_1)|1\leq i\leq n\}$

Suppose P_n^+ admits ATC into q parts. Then

$$|E(P_n^+)| = \frac{q(q+1)}{2}$$

$$2n-1 = \frac{q(q+1)}{2}$$

$$4n = q^2 + q + 2$$

Since $\frac{q(q+1)}{2}$ is odd, q or (q+1) cannot be the multiple of 4. So we must have q ≡ 1 or 2 (mod 4). Suppose ifq ≡ 3 (mod4) then (q+1) becomes a multiple of 4, which is a contradiction.

Thus q ≡ 1 or 2 (mod 4).

Conversely, suppose 4n = q^2 + q + 2

$$4n-2 = q^2 + q$$

$$2(2n-1) = q(q+1)$$

$$2n-1 = \frac{q(q+1)}{2}$$

$$|E(P_n^+)| = \frac{q(q+1)}{2}$$

Thus $^+$ can be decomposed into q parts and the ascending tree cover is given as follows:

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Case (i): $q \equiv 1 \pmod 4$.

(i.e) q = 4k + 1 for some positive integer k.

Let $T_1 = \{(u_1, v_1)\}$

$T_2 = \{(u_1, u_2), (u_2, v_2)\}$

$T_3 = \{(u_2, u_3, u_4), (u_3, v_3)\}$

$T_4 = \{(u_4, u_5, u_6), (u_4, v_4), (u_5, v_5)\}$

$T_5 = \{(u_6, u_7, u_8), (u_6, v_6), (u_7, v_7), (u_8, v_8)\}$

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$T_{4k-2} = \{(u_i u_{i+1}) \mid n - 8k \le i \le n - 6k - 1\} \cup \{(u_i, v_i) \mid n - 8k + 1 \le i \le n - 6k\}$

$T_{4k-1} = \{(u_i, u_{i+1}) \mid n - 6k \le i \le n - 4k - 1\} \cup \{(u_i, v_i) \mid n - 6k + 1 \le i \le n - 4k - 1\}$

$T_{4k} = \{(u_i, u_{i+1}) \mid n - 4k \le i \le n - 2k - 1\} \cup \{(u_i, v_i) \mid n - 4k \le i \le n - 2k - 1\}$

$T_{4k+1} = \{(u_i, u_{i+1}) \mid n - 2k \le i \le n - 1\} \cup \{(u_i, v_i) \mid n - 2k \le i \le n\}$

Thus $\{T_1, T_2, T_3, \ldots\ldots T_{4k+1} = q\}$ is an ATC for P_n^+.

Case (ii) $q \equiv 2 \pmod 4$

(i.e) q = 4k + 2 for some positive integer k.
Let $T_1 = \{(u_1, v_1)\}$
$T_2 = \{(u_1, u_2), (u_2, v_2)\}$
$T_3 = \{(_2, u_3, u_4), (u_3, v_3)\}$
$T_4 = \{(u_4, u_5, u_6), (u_4, v_4), (u_5, v_5)\}$
$T_5 = \{(u_6, u_7, u_8), (u_6, v_6), (u_7, v_7), (u_8, v_8)\}$

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$T_{4k-1} = \{(u_i, u_{i+1}) \mid n - 8k - 1 \le i \le n - 6k - 2\} \cup \{(u_i, v_i) \mid n - 8k \le i \le n - 6k - 2\}$

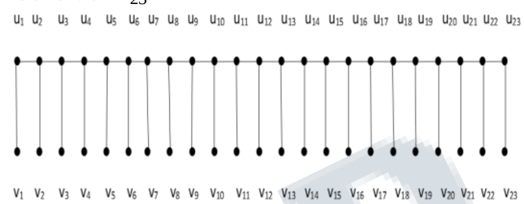
$T_{4k} = \{(u_i, u_{i+1}) \mid n - 6k - 1 \le i \le n - 4k - 2\} \cup \{(u_i, v_i) \mid n - 6k - 1 \le i \le n - 4k - 2\}$

$T_{4k+1} = \{(u_i, u_{i+1}) \mid n - 4k - 1 \le i \le n - 2k - 1\} \cup \{(u_i, v_i) \mid n - 4k - 1 \le i \le n - 2k - 1\}$

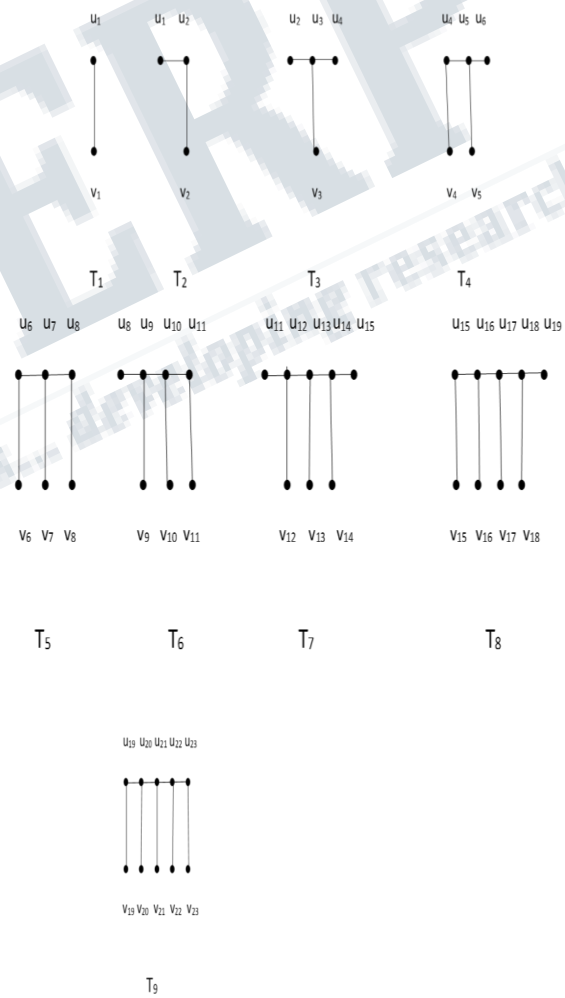
$T_{4k+2} = \{(u_i, u_{i+1}) \mid n - 2k - 1 \le i \le n - 1\} \cup \{(u_i, v_i) \mid n - 2k \le i \le n\}$

Thus $\{T_1, T_2, T_3, \ldots\ldots T_{4K+1} = q\}$ is an ATC for P_n^+.

Example:
Consider P_{23}^+



P_{23}^+



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Theorem 2.4

C_n^+, n ≥ 3 admits ATC into q parts iff q≡ 0 or 3 (mod 4) and $q^2 + q - 4n = 0$.

Proof:

Let V(C_n^+)={u_1,u_2, ... , u_n, v_1,v_2,..,v_n}

E(C_n^+)={(u_i,u_{i+1})|1≤i≤n-1}U {(u_i,v_i)|1≤ i ≤ n}U (u_1 , u_n)

Suppose C_n^+admits ATC into q parts .Then,

$$|E(C_n^+)| = \frac{q(q+1)}{2}$$

$$2n = \frac{q(q+1)}{2}$$

$$4n = q(q+1)$$

(i.e) q^2 + q – 4n = 0.

Since $\frac{q(q+1)}{2}$ is even, q or q+1 be the multiple of 4. so we must have q ≡ 0 or 3 (mod 4). Suppose if q ≡ 1(mod 4) then (q + 1) becomes not multiple of 4 , which is contradiction.

q ≡ 0 or 3 (mod 4)

Conversely, Suppose q^2 + q – 4n = 0

$q^2 + q = 4n$

$$2n = \frac{q(q+1)}{2}$$

$$|E(C_n^+)| = \frac{q(q+1)}{2}$$

Thus C_n^+ can be decomposed into q parts and ATC is given as follows:-

Case (i) q ≡ 0 (mod 4).

(i.e) q = 4k for some positive integer k.

Let T_1 = {(u_1 , v_1)}

T_2 = {(u_1 , u_2), (u_2 , v_2)}

T_3 = {(u_2 , u_3 , u_4), (u_3 ,v_3)}

T_4 = {(u_4 , u_5 , u_6), (u_4 , v_4), (u_5 , v_5)}

T_5 = {(u_6 , u_7 , u_8), (u_6 , v_6), (u_7 , v_7),(u_8 , v_8)}

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T_{4k-1}={(u_i , u_{i+1})|n - 4k+1 ≤ i≤ n – 2k }U{(u_i , v_i) | n – 4k + 2 ≤ i ≤ n - 2k }

T_{4k} ={(u_i , u_{i+1}) | n – 2k+1 ≤ i ≤ n – 1} U {(u_i , v_i) | n – 2k+1 ≤ i ≤ n }

Case (ii) q ≡ 0 (mod 4).

(i.e) q = 4k for some positive integer k.

Let T_1 = {(u_1 , v_1)}

T_2 = {(u_1 , u_2) , (u_2 , v_2)}

T_3 = {($_2$, u_3 , u_4), (u_3 , v_3)}

T_4 = {(u_4 , u_5 , u_6), (u_4 , v_4), (u_5,v_5)}

T_5 = {(u_6 , u_7 , u_8), (u_6 , v_6), (u_7 , v_7),(u_8 , v_8)}

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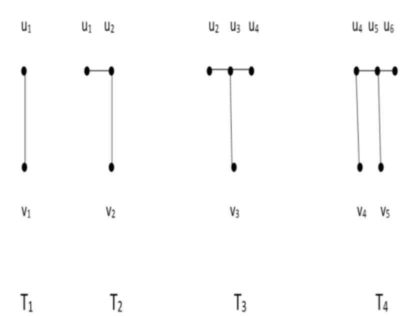
T_{4k+2} ={(u_i , u_{i+1}) | n–4k-2 ≤ i ≤ n -2k-2} U {(u_i , v_i) | n – 4k -1 ≤ i ≤ n -2k-1}

T_{4k+3} ={(u_i , u_{i+1}) | n – 2k-1 ≤ i ≤ n – 1} U (u_n , u_1)U{ (u_i ,v_i) | n – 2k ≤ i ≤ n }

Thus {T_1, T_2 , T_3……..T_{4k+3} = q} is an ATC for C_n^+.

Example:

Consider C_{18}^+



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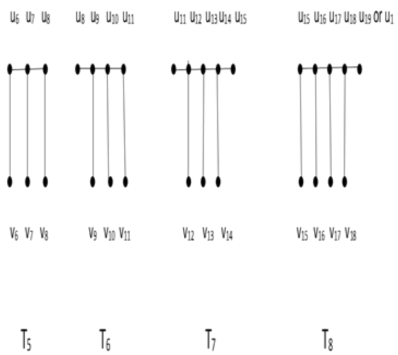
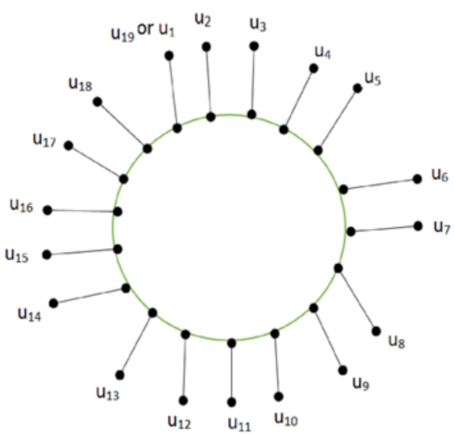


Consider C_{19}^+



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