

On Contra $b^*\hat{g}$ -continuous functions in Topological Spaces

[¹] K.Bala Deepa Arasi, [²] M.Mari Vidhya

[¹] Assistant Professor of Mathematics, A.P.C.Mahalaxmi College for Women, Thoothukudi, TN, India.

[²] M.Phil Scholar, St.Mary's College (Autonomous), Thoothukudi, TN, India.

Abstract:-- In this paper a new class of function called contra $b^*\hat{g}$ -continuous function is introduced and its properties are studied. Some characterization and several properties concerning contra $b^*\hat{g}$ -continuity are obtained.

Keywords:-- $b^*\hat{g}$ -closed sets, $b^*\hat{g}$ -continuous, contra $b^*\hat{g}$ -continuous.

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1. INTRODUCTION

In 1996, Dontchev[6] introduced and investigated a new notion of continuity called contra-continuity. Following this, many authors introduced many types of new generalizations of contra-continuity called as contra α -continuity, contra semi-continuity [5], contra gs-continuity [4], contra gb-continuity[13], contra $b\hat{g}$ -continuity[12] and so on and they investigated their properties. In 2015, K.Bala Deepa Arasi and G.Subasini introduced $b^*\hat{g}$ -closed sets[2] in Topological spaces. In 2017, we introduced $b^*\hat{g}$ -continuous functions and $b^*\hat{g}$ - open maps [3] in Topological spaces.

In this paper, we introduce and investigate some of the properties of contra $b^*\hat{g}$ -continuous function and we obtain some of its characterization.

2.PRELIMINARIES

Throughout this paper (X,τ) (or simply X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ) , $Cl(A)$, $Int(A)$ and A^c denote the closure of A , interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1: A subset A of a topological space (X,τ) is called

1. a semi-open set[4] if $A \subseteq Cl(Int(A))$.
2. an α -open set[7] if $A \subseteq Int(Cl(Int(A)))$.
3. a b-open set[1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.
4. a regular open set[11] if $A = Int(Cl(A))$.

The complement of a semi-open (resp. α -open, b-open, regular open) set is called semi-closed(resp. α -closed, b-closed, regular closed) set.

The intersection of all semi-closed (resp. α -closed, b-closed, regular closed) sets of X containing A is called the semi-closure (resp. α -closure, b-closure, regular closure) of A and

is denoted by $sCl(A)$ (resp. $\alpha Cl(A)$, $bCl(A)$, $rCl(A)$). The family of all semi-open (resp. α -open, b-open, regular open) subsets of a space X is denoted by $sO(X)$ (resp. $\alpha O(X)$, $bO(X)$, $rO(X)$).

Definition 2.2: A subset A of a topological space (X) is called

1. a gs-closed set[4] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
2. a gb-closed set[13] if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .
3. a $b\hat{g}$ -closed set[12] if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in X .
4. a $b^*\hat{g}$ -closed set[2] if $b^*Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in X .

The complement of a gs-closed (resp.gb-closed, $b\hat{g}$ -closed, $b^*\hat{g}$ -closed) set is called gs-open (resp.gb-open, $b\hat{g}$ -open, $b^*\hat{g}$ -open) set.

Definition 2.3: A space (X) is called a $T_{b^*\hat{g}}$ -space[2], if every $b^*\hat{g}$ -closed set in X is closed.

Definition 2.4: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a

1. $b^*\hat{g}$ -continuous map[3] if $f^{-1}(V)$ is $b^*\hat{g}$ -closed in (X, τ) for every closed set V in (Y, σ) .
2. $b^*\hat{g}$ -irresolute map[3] if $f^{-1}(V)$ is $b^*\hat{g}$ -closed in (X, τ) for every $b^*\hat{g}$ -closed set V in (Y, σ) .

Definition 2.5: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a

1. contra continuous map[6] if $f^{-1}(V)$ is closed in (X, τ) for every open set V in (Y, σ) .
2. contra semi-continuous map[5] if $f^{-1}(V)$ is semi-closed in (X, τ) for every open set V in (Y, σ) .

3. contra α -continuous map[7] if $f^{-1}(V)$ is α -closed in (X, τ) for every open set V in (Y, σ) .
4. contra gs-continuous map[4] if $f^{-1}(V)$ is gs-closed in (X, τ) for every open set V in (Y, σ) .
5. contra gb-continuous map[13] if $f^{-1}(V)$ is gb-closed in (X, τ) for every open set V in (Y, σ) .
6. contra $b\hat{g}$ -continuous map[12] if $f^{-1}(V)$ is $b\hat{g}$ -closed in (X, τ) for every open set V in (Y, σ) .

Definition 2.6:[12] A space (X, τ) is said to be locally indiscrete if every open subset of X is closed in X .

Definition 2.7:[4] A topological space (X, τ) is said to be Urysohn space if for each pair of distinct points x and y in X , there exists two open sets U and V in X such that $x \in U, y \in V$ and $Cl(U) \cap Cl(V) = \Phi$.

Definition 2.8:[4] For a map $f: X \rightarrow Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

3. CONTRA $b^*\hat{g}$ -CONTINUOUS FUNCTIONS

We introduce the following definition.

Definition 3.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called contra $b^*\hat{g}$ -continuous if $f^{-1}(V)$ is $b^*\hat{g}$ -closed in (X, τ) for every open set V in (Y, σ) .

Example 3.2: Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{X, \phi, \{a\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. $b^*\hat{g}C(X) = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here, the inverse image of $O(Y) \{a\}$ and $\{a, b\}$ are $\{b\}, \{c\}$ and $\{a, b\}$ which are $b^*\hat{g}C(X)$. Hence, f is contra $b^*\hat{g}$ -continuous.

Theorem 3.3:

- a) Every contra continuous function is contra $b^*\hat{g}$ -continuous.
- b) Every contra α -continuous function is contra $b^*\hat{g}$ -continuous.
- c) Every contra semi-continuous function is contra $b^*\hat{g}$ -continuous.

Proof:

- a) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra continuous function. Let V be any open set in (Y, σ) . Since f is contra continuous, $f^{-1}(V)$ is closed set in (X, τ) . By proposition 3.4 in [2], $f^{-1}(V)$ is $b^*\hat{g}$ -closed in (X, τ) . Hence, f is contra $b^*\hat{g}$ -continuous function.

- b) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra α -continuous function. Let V be any open set in (Y, σ) . Since f is contra α -continuous, $f^{-1}(V)$ is α -closed set in (X, τ) . By proposition 3.6 in [2], $f^{-1}(V)$ is $b^*\hat{g}$ -closed in (X, τ) . Hence, f is contra $b^*\hat{g}$ -continuous function.
- c) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra semi-continuous function. Let V be any open set in (Y, σ) . Since f is contra semi-continuous, $f^{-1}(V)$ is semi-closed set in (X, τ) . By proposition 3.6 in [2], $f^{-1}(V)$ is $b^*\hat{g}$ -closed in (X, τ) . Hence, f is contra $b^*\hat{g}$ -continuous function.

The following examples show that the converse of the above proposition need not be true.

Example 3.4:

- a) Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{X, \phi, \{a\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. $C(X) = \{X, \phi, \{b, c\}\}$; $b^*\hat{g}C(X) = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Here, the inverse image of $O(Y) \{a\}$ and $\{b, c\}$ are $\{b\}$ and $\{a, c\}$ which are $b^*\hat{g}C(X)$ but not $C(X)$. Hence, f is contra $b^*\hat{g}$ -continuous but not contra continuous.
- b) Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. $\alpha C(X) = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\}$; $b^*\hat{g}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}\}$. Here, the inverse image of $O(Y) \{a\}$ and $\{a, b\}$ are $\{b\}$ and $\{b, c\}$ which are $b^*\hat{g}C(X)$ but not $\alpha C(X)$. Hence, f is contra $b^*\hat{g}$ -continuous but not contra α -continuous.
- c) Let $X = Y = \{a, b, c\}$ with topologies $\tau = \{X, \phi, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = c, f(c) = b$. $sC(X) = \{X, \phi, \{a\}\}$; $b^*\hat{g}C(X) = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Here, the inverse image of $O(Y) \{a\}$ and $\{a, b\}$ are $\{a\}$ and $\{a, c\}$ which are $b^*\hat{g}C(X)$ but not $sC(X)$. Hence, f is contra $b^*\hat{g}$ -continuous but not contra semi-continuous.

Theorem 3.5:

- a) Every contra $b^*\hat{g}$ -continuous function is contra gs-continuous.

- b) Every contra $b^*\hat{g}$ -continuous function is contra gb-continuous.
- c) Every contra $b^*\hat{g}$ -continuous function is contra $b\hat{g}$ -continuous.

Proof:

- a) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra $b^*\hat{g}$ -continuous. Let V be any open set in (Y, σ) . Since f is contra $b^*\hat{g}$ -continuous, $f^{-1}(V)$ is $b^*\hat{g}$ -closed set in (X, τ) . By proposition 3.8 in [2], $f^{-1}(V)$ is gs-closed in (X, τ) . Hence, f is contra gs-continuous.
- b) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra $b^*\hat{g}$ -continuous. Let V be any open set in (Y, σ) . Since f is contra $b^*\hat{g}$ -continuous, $f^{-1}(V)$ is $b^*\hat{g}$ -closed set in (X, τ) . By proposition 3.12 in [2], $f^{-1}(V)$ is gb-closed in (X, τ) . Hence, f is contra gb-continuous.
- c) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra $b^*\hat{g}$ -continuous. Let V be any open set in (Y, σ) . Since f is contra $b^*\hat{g}$ -continuous, $f^{-1}(V)$ is $b^*\hat{g}$ -closed set in (X, τ) . By proposition 3.10 in [2], $f^{-1}(V)$ is $b\hat{g}$ -closed in (X, τ) . Hence, f is contra $b\hat{g}$ -continuous.

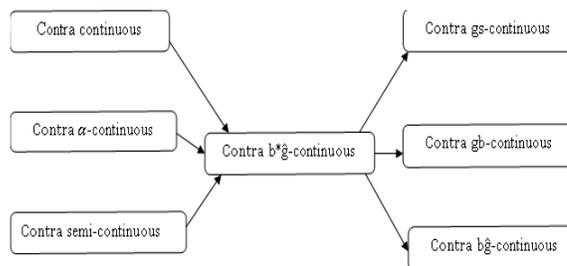
The converse of the above theorem need not be true as shown in the following example.

Example 3.6:

- a) Let $X = Y = \{a,b,c\}$ with topologies $\tau = \{X, \phi, \{a\}, \{b,c\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{c\}, \{b,c\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. $gsC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$; $b^*\hat{g}C(X) = \{X, \phi, \{a\}, \{b,c\}\}$. Here, the inverse image of $O(Y)$ $\{b\}, \{c\}$ and $\{b,c\}$ are $\{c\}$ $\{a\}$ and $\{a,c\}$ which are $gsC(X)$ but not $b^*\hat{g}C(X)$. Hence, f is contra gs-continuous but not contra $b^*\hat{g}$ -continuous.
- b) Let $X = Y = \{a,b,c\}$ with topologies $\tau = \{X, \phi, \{c\}, \{a,b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b,c\}\}$. Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. $gbC(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$; $b^*\hat{g}C(X) = \{X, \phi, \{c\}, \{a,b\}\}$. Here, the inverse image of $O(Y)$ $\{a\}$ and $\{b,c\}$ are $\{b\}$ and $\{a,c\}$ which are $gbC(X)$ but not $b^*\hat{g}C(X)$. Hence, f is contra gb-continuous but not contra $b^*\hat{g}$ -continuous.
- c) Let $X = Y = \{a,b,c\}$ with topologies $\tau = \{X, \phi, \{a\}, \{b,c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$.

Define $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b$. $b\hat{g}C(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$; $b^*\hat{g}C(X) = \{X, \phi, \{a\}, \{b,c\}\}$. Here, the inverse image of $O(Y)$ $\{a\}, \{c\}, \{a,c\}$ and $\{b,c\}$ are $\{c\}, \{b\}, \{b,c\}$ and $\{a,b\}$ which are $b\hat{g}C(X)$ but not $b^*\hat{g}C(X)$. Hence, f is contra $b\hat{g}$ -continuous but not contra $b^*\hat{g}$ -continuous.

Remark 3.7: The following diagram shows the relationships of contra $b^*\hat{g}$ -continuous function with the other known existing functions. $A \rightarrow B$ represents A implies B but not conversely.



Theorem 3.8: The following are equivalent for a function $f: (X, \tau) \rightarrow (Y, \sigma)$.

- a) f is contra $b^*\hat{g}$ -continuous function
- b) For every closed subset F of $Y, f^{-1}(F)$ is $b^*\hat{g}$ -open in X
- c) For each $x \in X$ and each closed subset F of Y with $f(x) \in F$ there exists a $b^*\hat{g}$ -open set U of X with $x \in U, f(U) \subseteq F$.

Proof:

- (a) \implies (b): Let F be any closed set in Y . Then F^c is an open set in Y . Since f is contra $b^*\hat{g}$ -continuous, $f^{-1}(F^c)$ is $b^*\hat{g}$ -closed set in X . Then $[f^{-1}(F)]^c$ is $b^*\hat{g}$ -closed set in X . Therefore $f^{-1}(F)$ is $b^*\hat{g}$ -open in X .
- (b) \implies (a): Let F be any open set in Y . Then F^c is a closed set in Y . By (b), $f^{-1}(F^c)$ is $b^*\hat{g}$ -open set in X . Then $[f^{-1}(F)]^c$ is $b^*\hat{g}$ -open set in X . So $f^{-1}(F)$ is $b^*\hat{g}$ -closed set in X . Therefore f is contra $b^*\hat{g}$ -continuous function.
- (b) \implies (c): Let F be any closed subset of Y and let $f(x) \in F$ where $x \in X$. Then by (b), $f^{-1}(F)$ is $b^*\hat{g}$ -open in X . Also, $x \in f^{-1}(F)$. Take $U = f^{-1}(F)$. Then U is a $b^*\hat{g}$ -open set containing x and $f(U) \subseteq F$.
- (c) \implies (b):

Let F be any closed subset of Y . If $x \in f^{-1}(F)$ then $f(x) \in F$. By (c), there exists a $b^*\hat{g}$ -open set U_x of X with $x \in U_x$ such that $f(U_x) \subseteq F$. Then $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$. Hence, $f^{-1}(F)$ is $b^*\hat{g}$ -open in X .

Theorem 3.9: If X is $T_{b^*\hat{g}}$ - space, then for the function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent.

- a) f is contra continuous function
- b) f is contra $b^*\hat{g}$ -continuous function.

Proof:

(a) \Rightarrow (b):

Let V be any open set in Y . Since f is contra continuous, $f^{-1}(V)$ is closed in X . From [2] proposition 3.4, $f^{-1}(V)$ is $b^*\hat{g}$ -closed in X . Therefore, f is contra $b^*\hat{g}$ -continuous.

(b) \Rightarrow (a):

Let V be any open set in Y . Since f is contra $b^*\hat{g}$ -continuous, $f^{-1}(V)$ is $b^*\hat{g}$ -closed in X . Also, since X is $T_{b^*\hat{g}}$ - space, $f^{-1}(V)$ is closed in X . Therefore, f is contra continuous.

Theorem 3.10: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function then the following statements are equivalent,

- a) f is $b^*\hat{g}$ -continuous function
- b) For each point $x \in X$ and each open set V of Y with $f(x) \in V$, there exists a $b^*\hat{g}$ -open set U of X such that $x \in U, f(U) \subseteq V$.

Proof:

(a) \Rightarrow (b):

Let V be any open set in Y and $f(x) \in V$, then $x \in f^{-1}(V)$. Since f is $b^*\hat{g}$ -continuous, $f^{-1}(V)$ is $b^*\hat{g}$ -open in X . Let $U = f^{-1}(V)$, then $x \in U$ and $f(U) \subseteq V$.

(b) \Rightarrow (a):

Let V be any open set in Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. From (b), there exists a $b^*\hat{g}$ -open set U_x of X such that $x \in U_x \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup\{U_x\}$. Then $f^{-1}(V)$ is $b^*\hat{g}$ -open in X . Hence, f is $b^*\hat{g}$ -continuous.

Theorem 3.11: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $b^*\hat{g}$ -continuous and Y is regular, then f is $b^*\hat{g}$ -continuous.

Proof: Let $x \in X$ and V be an open set in Y with $f(x) \in V$. Since Y is regular, there exists an open set W in Y such that $f(x) \in W$ and $\text{Cl}(W) \subseteq V$. Since f is contra $b^*\hat{g}$ -continuous and $\text{Cl}(W)$ is a closed subset of Y with $f(x) \in \text{Cl}(W)$. By theorem 3.8, there exist a $b^*\hat{g}$ -open set U of X with $x \in U$ such that $f(U) \subseteq \text{Cl}(W)$. That is, $f(U) \subseteq V$. By theorem 3.10, f is $b^*\hat{g}$ -continuous.

Definition 3.12: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $b^*\hat{g}$ -continuous if $f^{-1}(V)$ is closed in (X, τ) for every $b^*\hat{g}$ -closed set V in (Y, σ) .

Example 3.13: Let $X=Y=\{a,b,c\}$ with topologies $\tau = \{X, \Phi, \{a\}, \{c\}, \{a,c\}, \{b,c\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{b,c\}\}$. Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = c, f(c) = b$. $b^*\hat{g}\text{-}C(Y) = \{Y, \Phi, \{a\}, \{b,c\}\}$; $C(X) = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$. Here the inverse image of $b^*\hat{g}$ -closed sets $\{a\}$ and $\{b,c\}$ in (Y, σ) are $\{a\}$ and $\{b,c\}$ which are closed in (X, τ) . Hence, f is strongly $b^*\hat{g}$ -continuous.

Definition 3.14: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be perfectly $b^*\hat{g}$ -continuous if $f^{-1}(V)$ is clopen in (X, τ) for every $b^*\hat{g}$ -closed set V in (Y, σ) .

Example 3.15: Let $X=Y=\{a,b,c\}$ with topologies $\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$ and $\sigma = \{Y, \Phi, \{c\}, \{a,b\}\}$. Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c, f(b) = a, f(c) = b$. $b^*\hat{g}\text{-}C(Y) = \{Y, \Phi, \{c\}, \{a,b\}\}$; $\text{ClO}(X) = \{X, \Phi, \{b\}, \{a,c\}\}$. Here the inverse image of $b^*\hat{g}$ -closed sets $\{c\}$ and $\{a,b\}$ in (Y, σ) are $\{b\}$ and $\{a,c\}$ which are clopen in (X, τ) . Hence, f is perfectly $b^*\hat{g}$ -continuous.

Definition 3.16: A topological space (X, τ) is said to be $b^*\hat{g}$ -Hausdorff (or $b^*\hat{g}\text{-}T_2$ space) if for each pair of distinct points x and y in X , there exists $b^*\hat{g}$ -open subsets U and V of X containing x and y respectively such that $U \cap V = \Phi$.

Example 3.17: Let $X=\{a,b,c\}$ with topology $\tau = \{X, \Phi, \{b\}, \{c\}, \{b,c\}\}$; $b^*\hat{g}\text{-}O(X) = \{X, \Phi, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$. Clearly, (X, τ) is $b^*\hat{g}$ -Hausdorff space.

Theorem 3.18: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be surjective, closed and contra $b^*\hat{g}$ -continuous. If X is $T_{b^*\hat{g}}$ -space, then Y is locally indiscrete.

Proof: Let V be any open set in (Y, σ) . Since f is contra $b^*\hat{g}$ -continuous, $f^{-1}(V)$ is $b^*\hat{g}$ -closed in (X, τ) . Also, since X is $T_{b^*\hat{g}}$ -space, $f^{-1}(V)$ is closed in (X, τ) . By hypothesis, f is closed and surjective, $f(f^{-1}(V)) = V$ is closed in (Y, σ) . Hence, Y is locally indiscrete.

Theorem 3.19: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is continuous and (X, τ) is locally indiscrete space, then f is contra $b^*\hat{g}$ -continuous.

Proof: Let V be any open set in (Y, σ) . Since f is continuous, $f^{-1}(V)$ is open in (X, τ) . Since X is locally indiscrete, $f^{-1}(V)$ is closed in (X, τ) . By proposition 3.4 in [2], $f^{-1}(V)$ is $b^*\hat{g}$ -closed in (X, τ) . Hence, f is contra $b^*\hat{g}$ -continuous.

Theorem 3.20: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $b^*\hat{g}$ -continuous, injective and Y is Urysohn space, then the topological space X is $b^*\hat{g}$ -Hausdorff space.

Proof: Let x_1 and x_2 be two distinct points of X . Suppose $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since f is injective, $x_1 \neq x_2$ then $y_1 \neq y_2$. Since Y is Urysohn, there exist open sets V_1 and V_2 containing y_1 and y_2 respectively in Y such that $\text{Cl}(V_1) \cap \text{Cl}(V_2) = \Phi$. Since f is contra $b^*\hat{g}$ -continuous and by theorem 3.10, there exists $b^*\hat{g}$ -open sets U_1 and U_2 containing x_1 and x_2 respectively in X such that $(U_1) \subseteq \text{Cl}(V_1)$ and $f(U_2) \subseteq \text{Cl}(V_2)$. Since $\text{Cl}(V_1) \cap \text{Cl}(V_2) = \Phi$, $U_1 \cap U_2 = \Phi$. Hence, X is $b^*\hat{g}$ -Hausdorff space.

Theorem 3.21: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function and $g: X \rightarrow X \times Y$ be a graph function of f defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra $b^*\hat{g}$ -continuous, then f is contra $b^*\hat{g}$ -continuous.

Proof: Let V be a closed subset of Y . Then $X \times V$ is a closed subset of $X \times Y$. Since g is contra $b^*\hat{g}$ -continuous, $g^{-1}(X \times V)$ is $b^*\hat{g}$ -open subset of X . Also, $g^{-1}(X \times V) = f^{-1}(V)$ which is $b^*\hat{g}$ -open subset of X . Hence, f is contra $b^*\hat{g}$ -continuous.

Definition 3.22: A space X is said to be locally $b^*\hat{g}$ -indiscrete if every $b^*\hat{g}$ -open set of X is closed in X .

Example 3.23: Let $X = \{a, b, c\}$ with $\tau = \{X, \Phi, \{a\}, \{b, c\}\}$; $b^*\hat{g}\text{-O}(X) = \{X, \Phi, \{a\}, \{b, c\}\}$; $C(X) = \{X, \Phi, \{a\}, \{b, c\}\}$. Here, every $b^*\hat{g}$ -open sets in X are closed in X . Hence, X is locally $b^*\hat{g}$ -indiscrete space.

Theorem 3.24: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $b^*\hat{g}$ -continuous with X as locally $b^*\hat{g}$ -indiscrete, then f is continuous.

Proof: Let V be any open set in (Y, σ) . Since f is contra $b^*\hat{g}$ -continuous, $f^{-1}(V)$ is $b^*\hat{g}$ -closed in X . Since X is locally $b^*\hat{g}$ -indiscrete space, $f^{-1}(V)$ is open in X . Thus, f is continuous.

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