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A Note on SG – algebras

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Abstract:-- **In this paper we introduce the notion of SG –algebras as a generalization of BCK/BCI/Q/TM algebras and investigated some of the elementary properties by comparing with other algebras.**

Keywords:--**BCI algebra, BCH algebra, TM algebras, SG algebras.**

1. INTRODUCTION

Imai and Iseki (1966) introduced a class of abstract algebra known as BCK – algebra. At the same time , Iseki introduced another class of algebra, called BCI – algebra and investigated some of its properties . it is known that class of BCK – algebra is a proper sub class of the class of BCI – algebra. These two are the important classes logical algebras.Hu and Li (1985) introduced a wide class of abstract algebra namely,BCH – algebras and have shown that the class of BCI – algebras is a proper subclass of the class of BCH – algebras. J.Negger and Kim(1999) introduced the notion of d – algebras which is another useful generalization of BCK – algebras. At the same time, Jun,Roh and Kim (1998) introduced the new notion called BH – algebra, a generalization of BCH/BCI/BCK algebras.Ahn ,Neggers and Kim (20010 introduced Q – algebras which is generalization of BCH/BCI/BCK algebras. K.Megalai and A.Tamilarasi introduced a class of abstract algebras: TM – algebra, which is a generalization of Q/BCK/BCI/BCH – algebras. Motivated by this, we introduce the notion of SG – algebras which is the generalization of BCI/BCK / BCH algebras. We study the properties if SG – algebras. also we define the associative SG algebra and study some of its properties.

2.Preliminaries

A **Q – algebra** is a non – empty set X with a constant 0 but a binary operation $*$ satisfying the axioms:
i)$x * x = 0$ ii)$x * 0 = x$ iii) $(x * y) * z = (x * z) * y \; for \; all \; x, y, z \in X$.
A Q – algebra $(X,*,0)$ is called a **QS – algebra** if $(x * y) * (x * z) = z * y \; for \; all \; x, y, z \in X$.
For brevity we shall call X a QS – algebra unless otherwise specified. In X we define a binary relation $\leq$ by $x \leq y$ if and only if $x * y = 0$. Note that every QS – algebra is a Q – algebra.A **BCI – algebra** is an algebra $(X,*,0)$ of type (2,0) satisfying the following conditions:i)$(x * y) * (x * z) \leq z * y$ $ii)x * (x * y) \leq y$ $iii)x \leq x$ $iv)x \leq y$ and $y \leq x$ imply $x = y$ $v)x \leq 0$ implies $x = 0$, where $x \leq y$ is defined by $x * y = 0$ for all $x. y, z \in X$.A **BCK – algebra** is an algebra $(X,*,0)$ of type (0,2) satisfying the following conditions: i)$(x * y) * (x * z) \leq z * y$ ii)$x * (x * y) \leq y$ $iii)x \leq x$ iv)$x \leq y$ and $y \leq x$ imply $x = y$ v)$0 \leq x$ implies $x = 0$, where $x \leq y$ is defined by $x * y = 0$ for all $x. y, z \in X$.A **BH – algebra** is a non – empty set X with a constant 0 and a binary operation $*$ satisfying the conditions:i)$x * x = 0$ ii)$x * 0 = x$ iii)$x * y = 0$ and $y * x = 0$ imply $x = y$ for all $x. y, z \in X$.A **BCH – algebra** is an algebra $(X,*,0)$ of type (2,0) satisfying the following axioms: i)$x \leq x$ ii)$x \leq y$ and $y \leq x$ imply $x = y$ iii) $(x * y) * z = (x * z) * y \; for \; all \; x, y, z \in X$.
A **d – algebra** is a non – empty set X with a constant 0 and a binary operation $*$ satisfying the conditions: i)$x * x = 0$ ii) $0 * x = 0$ iii)$x * y = 0$ and $y * x = 0$ imply $x = y$ for all $x. y, z \in X$.A **BG – algebra** is a non – empty set X with a constant 0 and a binary operation $*$ satisfying the axioms: i)$x * x = 0$ ii)$x * 0 = x$ iii)s$(x * y) * (0 * y) = x$ for all $x. y, z \in X$.A **TM – algebra** $(X,*,0)$ is a non – empty set X with a constant 0 and a binary operation $*$ satisfying the axioms: i)$x * 0 = x$ for $x \in X$ $ii)(x * y) * (x * z) = z * y$ for all $x. y, z \in X$.

3.Main Results

Definition 3.1

A SG – algebra $(X,*,0)$ is a non – empty set X with a constant 0 and a binary operation $*$ satisfying the axioms:

i) $x * 0 = x$ for $x \in X$
ii) $x * (x * y) = y$ for all $x, y \in X$

In X we can define a binary relation $\leq$ by $x \leq y$ if and only if $x * y = 0$

Example 3.2

Let Z be the set of all integers and let $nZ = \{nx : x \in Z\}, n \in Z$. Then $(Z, -, 0)$ and $(nZ, -, 0)$ are SG – algebras.(where '-' is the usual subtraction)

Solution:

i) $x - 0 = x \; for \; all \; x \in Z$

ii) $x - (x - y) = x - x + y = y$ for all $x, y \in Z$

Hence $(Z, -, 0)$ is a SG – algebra.

i) $nx - 0 = nx$ for all $n, x \in Z$

ii) $nx - (nx - ny) = nx - nx + ny = ny$ for all $n, x, y \in Z$

Hence $(nZ, -, 0)$ is a SG – algebra.

Example 3.3

Let X = {0,1,2,3} be a set with cayley table

$*$	0	1	2	3
0	0	1	2	3
1	1	0	2	3
2	2	1	0	3
3	3	1	2	0

Then $(X,*,0)$ is a SG – algebra.

Proposition 3.4

If $(X,*,0)$ is a SG – algebra, then $x * x = 0$.

Proof

$x * x \; = \; x * (\; x * 0) = 0.$

Proposition 3.5

Let $(X,*,0)$ be a SG – algebra. Then for any $x . y, z \in X$

i) $x * 0 = 0$ implies $x = 0$

ii) $x *(x * (\; x * y)) \; = x * y$

iii) $(x * (x * y)) * y = 0$

iv) If $x * y = 0, y * x = 0$ then $x = y$

Proof

i) Given $x * 0 = 0$

But by Definition 3.1,$x * 0 = x$. Hence $x = 0$.

ii) $x *(x * (\; x * y)) = x * y$

iii) $(x * (x * y)) * y$

$= y * y$

$= 0$ (by proposition 3.4)

iv) If $x * y = 0$ then $x = x * 0 = x * (x * y) = y$

Similarly, if $y * x = 0$ then $y = y * 0 = y * (y * x) = x$

Theorem 3.6

Every SG – algebra is a BCI – algebra

Proof

Let $(X,*,0)$ be a SG – algebra.

i) To prove $(x * y) * (x * z) \leq z * y$

Consider $(x * y) * (x * z)$,

Replace y by $x * z$, then $\big(x * (x * z)\big) * (x * z) = z * (x * z) \qquad = z * y$

ii) To prove $x * (x * y) = y$

This is directly follows from definition

iii) To Prove $x \leq x$ (ie)to prove $x * x = 0$,

$x * x = x * (x * 0) = 0$

iv) To Prove $x \leq y \; and \; y \leq x \; implies \; x = y$

$x \leq y => x * y = 0$

Therefore $x * (x * y) = x * 0 = x$

But $x * (x * y) = y$

Hence $y = x$.

Similarly,$y \leq x => y * x = 0$

$y * (y * x) = y * 0 = y$

But $y * (y * x) = x$

Hence $x = y$.

v) To Prove $x \leq 0 => x = 0$

$x \leq 0 => x * 0 = 0$

$x * (x * x) = 0 => x = 0$

Hence SG – algebra is a generalization of BCI – algebra.

Theorem 3.7

Every SG – algebra is a BH – algebra

Proof

Let $(X,*,0)$ be a SG – algebra.

i) $x * x = 0$ is directly follows from Proposition 3.4.

ii) $x * 0 = x$ is directly follows from definition 3.1

iii) $x * y = 0 \; and \; y * x = 0 => x = y \; for \; all \; x, y \in X$,by proposition 3.5(iv).

Hence SG – algebra is a BH – algebra.

Theorem 3.8

Every SG – algebra is a BCH – algebra .

Proof

Let $(X,*,0)$ be a SG – algebra.

Every BCI – algebra is a BCH – algebra. But SG –algebra is a BCI – algebra

Hence SG – algebra is a BCH -algebra.

Theorem 3.9

Every Q – algebra is a SG – algebra

Proof

Every SG – algebra is a BCI – algebra

In BCI – algebra, condition (iii) for Q – algebra is satisfied.

Therefore SG – algebra is a Q – algebra.

Theorem 3.10

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Every SG – algebra is a BG – algebra

Proof

Let $(X,*,0)$ be a SG – algebra.

i) $x * x = 0$ follows from Proposition 3.4

ii) $x * 0 = x$ is directly follows from definition.

iii) To prove $x * y) * (0 * Y) = x$

In SG - algebra , $(x * y) * z = (x * z) * y$

Consider $0 * y = (x * x) * y = (x * y) * x$

Now $(x * y) * (0 * y) = (x * y) * \big((x * y) * x\big) = x$

Hence SG – algebra is a BG – algebra.

Theorem 3.11

Every BCK –algebra is a SG – algebra

Proof

$x * (x * y) = y$ is directly follows from Definition of BCK – algebra

$x * 0 = x * (x * x) = x$

Therefore BCK – algebra is a SG – algebra.

Remark 3.12

The converse is not true as shown in Example 3.3.Then $(X,*,0)$ is a SG – algebra, but not BCK – algebra, since$(0 * 1) * (0 * 2) \neq 2 * 1$.

Theorem 3.13

Every QS – algebra is a SG – algebra

Proof

Let $(X,*,0)$ be a QS – algebra.

Clearly, $x * 0 = x$ for all $x \in X$

To prove $x * (x * y) = y$

But we have $(x * y) * (x * z) = z * y$

Put $y = 0$, $(x * 0) * (x * z) = z * 0$

$x * (x * z) = z$.

Remark 3.14

The converse is not true as shown in Example 3.3.Then $(X,*,0)$ is a SG – algebra, but not QS – algebra, since$(0 * 1) * (0 * 2) \neq 2 * 1$.

Theorem3.15

Every TM – algebra is a SG – algebra

Proof

Let $(X,*,0)$ be a TM – algebra.

$x * 0 = x$ is directly follows

To Prove $x * (x * y) = y$

Since $(x * y) * (x * z) = z * y$

Put $y = 0$,$(x * 0) * (x * z) = z * 0$

$x * (x * z) = z$

Hence every TM – algebra is a SG – algebra.

Remark 3.16

The converse is not true as shown in Example 3.3.Then $(X,*,0)$ is a SG – algebra, but not TM– algebra, since$(0 * 1) * (0 * 2) \neq 2 * 1$.

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