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(γ,eD)-Number of Edge Added Graphs

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***Abstract:--* A subset S of V is called an edge detour set of G if every edge in G lies on a detour joining a pair of vertices of S. The edge detour number dn1(G) of G is the minimum order of its edge detour sets and any edge detour set of order dn1 is an edge detour basis. An edge detour dominating set is a subset S of V(G) which is both dominating and an edge detour set of G. The smallest cardinality of an edge detour dominating set of G is called the edge detour domination number of G. In this paper, it is found for the (γ,eD)-number of an edge added graphs of some known graphs such as path, cycle and complete graph.**

Keywords – Edge Detour, Edge detour domination, Edge detour domination of edge added.

1. INTRODUCTION

The concept of domination was introduced by Ore and Berge[6]. Let G be a finite, undirected connected graph with neither loops nor multiple edges. A subset D of V(G) is a dominating set of G if every vertex in V-D is adjacent to atleast one vertex in D. The minimum cardinality among all dominating sets of G is called the domination number γ(G) of G. We consider connected graphs with atleast two vertices. For basic definitions and terminologies, we refer Harary[1]. For vertices u and v in a connected graph G, the detour distance D(u,v) is the length of longest u-v path in G. A u-v path of length D(u,v) is called a u-v detour. A subset S of V(G) is called a detour set if every vertex in G lie on a detour joining a pair of vertices of S. The detour number dn(G) of G is the minimum order of a detour set and any detour set of order dn(G) is called a detour basis of G. These concepts were studied by Chartrand[3]. A subset S of V(G) is called an edge detour set of G if every edge in G lie on a detour joining a pair of vertices of S. The edge detour number $dn_1(G)$ of G is the minimum order of its edge detour sets and any edge detour set of order dn_1 is an edge detour basis. A graph G is called an edge detour graph if it has as edge detour set. Edge detour graph were introduced and studied by Santhkumaran and Athisayanathan[8].

The following results are from [4].

Theorem 1.1: The domination numbers of some standard graph are given as follows.

1. $\gamma(P_p) = \left\lceil \frac{p}{3} \right\rceil, p \geq 3$

2. $\gamma(C_p) = \left\lceil \frac{p}{3} \right\rceil, p \geq 3$

3. $\gamma(K_p) = \gamma(W_p) = \gamma(K_{1,n}) = 1$.

4. $\gamma(K_{m,n}) = 2$ if $m, n \geq 2$.

Theorem 1.2: A dominating set D of G is a minimal dominating set of G if and only if for every $v \in D$, there exists at least one vertex $w \in V-(D-\{v\})$ such that $N[w] \cap D = \{v\}$.

The following theorems were proved by A.P.Santhakumaran and S. Athisayanathan [8].

Theorem 1.3: For any edge detour graph G of order $p \geq 2$, $2 \leq dn_1(G) \leq p$.

Theorem 1.4: If G is an edge detour graph of order $p \geq 3$ such that {u,v} is an edge detour basis of G, then u and v are not adjacent.

Theorem 1.5: If T is a tree with k end vertices, then $dn_1(T) = k$.

Remark 1.6:

$$\left\lceil \frac{n-4}{3} \right\rceil + 2 = \begin{cases} \left\lceil \frac{n}{3} \right\rceil \text{ if } n \equiv 1 (\text{mod } 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 \text{ if otherwise} \end{cases}$$

The following are from Mahalakshmi.A, Palani.K and Somasundaram.S[5]

Theorem 1.7: K_P is an edge detour dominating graph and $\gamma_{eD}(K_P) = 3$ for $p \geq 3$.

Theorem 1.8: $\gamma_{eD}(K_{1,n}) = n$.

Theorem 1.9:

$$\gamma_{eD}(P_n) = \begin{cases} \left\lceil \frac{n-4}{3} \right\rceil + 2 \text{ if } n \geq 5 \\ 2 \text{ if } n = 2, 3 \text{ or } 4 \end{cases}$$

Theorem 1.10: For n>5,

$\gamma_{eD}(C_n) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Remark 1.11:

1. $\gamma_{eD}(G) \geq dn_1(G)$ and $\gamma_{eD}(G) \geq \gamma(G)$.
2. If the set of all pendant vertices of a graph G forms an edge detour dominating set G, then S is the unique minimum edge detour dominating set of G.
3. Every super set of an edge detour dominating set of G is an edge detour dominating set of G.

2 (γ,eD)-number of edge added graphs:

Theorem 2.1: If G' is the graph obtained from P_n ($n = 3k+1$) by adding a new vertex v to one of its vertices then, $\gamma_{eD}(G')=\gamma_{eD}(P_n)+1$.

Proof: Let $P_n = (v_1, v_2, \ldots, v_n)$.

Case (i): v is added to v_1 or v_n. Then, $G' \cong P_{n+1}$. As $n = 3k + 1$, $G' \cong P_{3k+2}$.

Therefore, $\gamma_{eD}(G') = \gamma_{eD}(P_{n+1}) = \gamma_{eD}(P_{3k+2}) = \left\lceil \frac{3\kappa + 2 - 4}{3} \right\rceil + 2 = \left\lceil (k-1) + \frac{1}{3} \right\rceil + 2 = k + 2 = (k+1)+1 = (\left\lceil \frac{3k+1-4}{3} \right\rceil + 2) + 1 = \gamma_{eD}(P_{3k+1})+1 = \gamma_{eD}(P_n) + 1$.

Therefore, $\gamma_{eD}(G') = \gamma_{eD}(P_n) + 1$.

Case (ii): v is added to an internal vertex.

Therefore, v becomes an end vertex of G'. Therefore, v ∊ every edge detour dominating set of G'. Since $n = 3k+1$, P_n has a unique edge detour dominating set, say S. Then, clearly, $S \cup \{v\}$ is a unique edge detour dominating set of G'. Therefore, $\gamma_{eD}(G') = \gamma_{eD}(P_n) + 1$. By cases of (i) and (ii), $\gamma_{eD}(G') = \gamma_{eD}(P_n) + 1$.

Theorem 2.2: If G' is the graph obtained from P_n ($n = 3k$) by adding a new vertex v to one of its vertices. Then,

$$\gamma_{eD}(G') = \begin{cases} \gamma_{eD}(P_n) \text{ if v is added to end vertices } v_1 \text{ or } v_n. \\ \gamma_{eD}(P_n) + 1 \text{ if v is added to an internal vertices.} \end{cases}$$

Proof: Let $P_n = (v_1, v_2, \ldots, v_n)$.

Case (i): v is added to v_1 or v_n . Then, $G' \cong P_{n+1}$. As $n = 3k$, $G' \cong P_{3k+1}$. Then,

$\gamma_{eD}(G') = \gamma_{eD}(P_{n+1}) = \gamma_{eD}(P_{3k+1}) = \left\lceil \frac{3k+1-4}{3} \right\rceil + 2 = (k-1)+2 = \left\lceil \frac{3(k-1)}{3} - \frac{1}{3} \right\rceil + 2 = \left\lceil \frac{3k-3-1}{3} \right\rceil + 2 = \gamma_{eD}(P_{3k}) = \gamma_{eD}(P_n)$. Therefore, $\gamma_{eD}(G') = \gamma_{eD}(P_n)$.

Case (ii): v is added to an internal vertex. Therefore, v becomes an end vertex of G'. Therefore, $v \in$ every edge detour dominating set of G'. Also, $S \cup \{v\}$ is an edge detour dominating set of G' if and only if S is an edge detour dominating set of P_n. Therefore, $\gamma_{eD}(G') = \gamma_{eD}(P_n) + 1$.

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Therefore, by cases (i) and (ii) $\gamma_{eD}(G') = \gamma_{eD}(P_n)$ or $\gamma_{eD}(P_n) + 1$.

Theorem 2.3: If G' is the graph obtained from the star graph $K_{1,n}$ by adding a new vertex v to one of its vertices then, $\gamma_{eD}(G') =$

$$\begin{cases} \gamma_{eD}(K_{1,n}) + 1 \text{ if v is added to the central vertex} \\ \gamma_{eD}(K_{1,n}) \text{ otherwise} \end{cases}$$

Proof: Let $V(K_{1,n}) = (v, v_1, v_2, \dots, v_n)$.

Case (i): v is added to the central vertex. Then, v is an end vertex of G'. Therefore, $v \in$ every edge detour dominating set of G'. Clearly, $S \cup \{v\}$ is the unique edge detour dominating set of G' whenever S is a edge detour dominating set of $K_{1,n}$ and vice-versa.

Therefore, $\gamma_{eD}(G') = \gamma_{eD}(K_{1,n}) + 1$.

Case (ii): v is added to an end vertex. Then, v becomes an end vertex of G' and the end vertex in which v is joined becomes an internal vertex of G'. Let it be v_i, for some i, $1 \le i \le n$. Therefore, $v \in$ every edge detour dominating set of G'. Then, $S' = S - \{v_i\} \cup \{v\}$ is the unique edge detour dominating set of G'. By Theorem 1.10, $K_{1,n}$ has a unique edge detour dominating set S (say). Therefore, $\gamma_{eD}(G') = |S'| = |S| = \gamma_{eD}(K_{1,n})$. Hence, $\gamma_{eD}(G') = \gamma_{eD}(K_{1,n})$ or $\gamma_{eD}(K_{1,n}) + 1$.

Theorem 2.4: If G' is the graph obtained from the cycle $C_n(n = 3k)$ by adding a new vertex v to one of its vertices then,

$$\gamma_{eD}(G') = \begin{cases} \gamma_{eD}(C_n) \text{ if } k = 1 \\ \gamma_{eD}(C_n) + 1 \text{ if } k \ge 2. \end{cases}$$

Proof: Let $C_n = (v_1, v_2, \dots, v_n, v_1)$.

Case 1: When k = 1, then the cycle is C_3 and $V(C_3) = \{v_1, v_2, v_3\}$. If v is added to any vertex of the cycle then, v becomes an end vertex of G'. Therefore, $v \in$ every edge detour dominating set of G'. Suppose the vertex v_1 is joined to v. Then, $S' = \{v, v_2, v_3\}$ is the unique edge detour dominating set of G'. Therefore, $\gamma_{eD}(G') = 3 = \gamma_{eD}(C_3)$.

Case 2: $k \ge 2$. The cycle is C_{3k} and $V(C_{3k}) = \{v_1, v_2, \dots, v_{3k}\}$. If v is added to any vertex of the cycle then, v becomes an end vertex of G'. Therefore, v belongs to every edge detour dominating set of G'. Name the vertex in which v is added as v_1. Then, $S_1 = \{v, v_3, v_6, \dots, v_{3k}\}$, $S_2 = \{v, v_2, v_5, \dots, v_{3k-1}\}$ and $S_3 = \{v, v_1, v_4, \dots, v_{3k-2}\}$ are some minimum edge detour dominating sets of G'. Therefore, $\gamma_{eD}(G') = k + 1 = \left\lceil \frac{3k}{3} \right\rceil + 1 = \gamma_{eD}(C_{3k}) + 1$.

Hence, in this case, $\gamma_{eD}(G') = \gamma_{eD}(C_n) + 1$.

Theorem 2.5: If G' is the graph obtained from the cycle $C_n(n = 3k + 1)$ by adding a new vertex v to one of its vertices

$$\text{then, } \gamma_{eD}(G') = \begin{cases} \gamma_{eD}(C_n) + 1 \text{ if } k = 2 \\ \gamma_{eD}(C_n) \text{ otherwise.} \end{cases}$$

Proof: Let $C_n = (v_1, v_2, \dots, v_n, v_1)$.

Case 1: When k = 2, the cycle is C_7 and $V(C_7) = \{v_1, v_2, \dots, v_7\}$. If v is added to any vertex of the cycle then, v becomes an end vertex of G'. Therefore, $v \in$ every edge detour dominating set of G'. Suppose the vertex v is added as v_1. Then, $S_1 = \{v, v_1, v_4, v_7\}$, $S_2 = \{v, v_2, v_4, v_6\}$ and $S_3 = \{v, v_3, v_5, v_6\}$ are some minimum edge detour dominating set of G'. Therefore, $\gamma_{eD}(G') = 4 = 3 + 1 = \left\lceil \frac{7}{3} \right\rceil + 1 = \gamma_{eD}(C_7) + 1$.

Hence, in this case, $\gamma_{eD}(G') = \gamma_{eD}(C_n) + 1$.

Case 2: When $k \ne 2$, the cycle is C_{3k+1} and $V(C_{3k+1}) = \{v_1, v_2, \dots, v_{3k+1}\}$. If v is added to any vertex of the cycle then, v becomes an end vertex of G'. Therefore, v belongs to every edge detour dominating set of G'. Name the vertex in which v is joined to v_1.

Then, $S' = \{v, v_3, v_6, \dots, v_{n-1}\} = \{v, v_3, v_6, \dots, v_{3k}\}$ is the unique edge detour dominating sets of G'. Therefore, $\gamma_{eD}(G')$

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$= k+1= \left\lceil \frac{3k+1}{3} \right\rceil + 1 = \gamma_{eD}(C_{3k+1}) + 1$. Hence, in this case $\gamma_{eD}(G') = \gamma_{eD}(C_n)$.

Theorem 2.6: If G' is the graph obtained from the cycle C_n(n = 3k+2) by adding a new vertex v to one of its vertices then,

$$\gamma_{eD}(G') = \begin{cases} \gamma_{eD}(C_n) \text{ if } k=1 \\ \gamma_{eD}(C_n)+1 \text{ if } k>1. \end{cases}$$

Proof: Let $C_n = (v_1, v_2, \ldots, v_n, v_1)$.

Case 1: When k = 1, the cycle is C_5 and $V(C_5) = \{v_1, v_2, v_3, v_4, v_5\}$. If v is added to any vertex of the cycle then, v becomes an end vertex of G'. Therefore, v ∊ every edge detour dominating set of G'. Suppose the vertex to which v is added is v_1.

Then, $S_1 = \{v, v_3, v_5\}$, $S_2 = \{v, v_2, v_4\}$ are some minimum edge detour dominating set of G'. Therefore, $\gamma_{eD}(G') = 3 = \gamma_{eD}(C_5)$. Hence, $\gamma_{eD}(G') = \gamma_{eD}(C_n)$, n = 3k+2 if k = 1.

Case 2: k >1.

Here, the cycle is C_{3k+2} and $V(C_{3k+2}) = \{v_1, v_2, \ldots, v_{3k+2}\}$. If v is added to any vertex of the cycle then, v becomes an end vertex of G'. Therefore, v belongs to every edge detour dominating set of G'. Name the vertex in which v is added as v_1. Then, $S_1 = \{v, v_3, v_6, \ldots, v_{3k}, v_{3k+2}\}$, $S_2 = \{v, v_2, v_5, \ldots, v_{3k-1}, v_{3k+1}\}$ and $S_3 = \{v, v_1, v_4, \ldots, v_{3k-2}, v_{3k+1}\}$ are edge detour dominating sets of G'. Therefore, $\gamma_{eD}(G') = k + 1 + 1 = \left\lceil \frac{3k+2}{3} \right\rceil + 1 = \gamma_{eD}(C_{3k+2}) + 1$. Hence, $\gamma_{eD}(G') = \gamma_{eD}(C_n) + 1$, n=3k+2 if $k \geq 2$.

Theorem 2.7: If G' is the graph obtained from the complete graph K_n by adding a new vertex v to one of its vertices then,

$$\gamma_{eD}(G') = \begin{cases} 2 \text{ if } n=2 \\ 3 \text{ if } n \geq 3. \end{cases}$$

Proof: Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$.

Case 1: When n = 2, then $G \cong K_2$.

If v is added to added to any of its vertices v_1 or v_2 then, v becomes an end vertex and $G' \cong P_3$. Therefore, by Theorem 1.10, $\gamma_{eD}(G') = \gamma_{eD}(P_3) = 2$.

Case 2: when $n \geq 3$.

If v is added to any vertex v_i, $1 \leq i \leq n$ of K_n, then v becomes an end vertex. Therefore, v belongs to every edge detour dominating set of G' and $S - \{v_i\} \cup \{v\}$ is a minimum edge detour dominating set of G' whenever S is a minimum edge detour dominating set of K_n.

Therefore, $\gamma_{eD}(G') = |S| = \gamma_{eD}(K_n)$. Hence, $\gamma_{eD}(G') = \gamma_{eD}(K_n)$. By Theorem 1.9,

$\gamma_{eD}(G') = 3$.

Theorem 2.8: Let G be a non-complete connected graph with a unique minimum edge detour dominating set D. Let G' be a graph obtained from G by adjoining an edge to a vertex of D. Then, $\gamma_{eD}(G) \leq \gamma_{eD}(G') \leq \gamma_{eD}(G) + 1$.

Proof: Let G' be a graph obtained from G by adjoining an edge to a vertex of D. Let S be the unique minimum edge detour dominating set of G. Suppose $V(G') = V(G) \cup \{u\}$ and $E(G) \cup \{uv\}$. Then, $S \cup \{u\}$ is an edge detour dominating set of G'.

Therefore, $\gamma_{eD}(G') \leq |S \cup \{u\}| = |S| + 1 = \gamma_{eD}(G) + 1$. Now, to prove $\gamma_{eD}(G) \leq \gamma_{eD}(G')$. Assume that, $\gamma_{eD}(G) > \gamma_{eD}(G')$. Let S' be a minimum edge detour dominating set of G'.

By our assumption $|S'| \leq \gamma_{eD}(G)-1$. Since u is an end vertex of G', $u \in S'$. Then, two cases arise.

Case 1: $v \in S'$. Here, $S'' = S' - \{u\}$ is an edge detour dominating set of G, and so

$\gamma_{eD}(G) \leq |S''| = |S' - \{u\}| \leq \gamma_{eD}(G) - 2$, which is a contradiction.

Case 2: $v \notin S'$. Define $S'' = (S' - \{u\}) \cup \{v\}$. Then, $|S''| = |S'| \leq \gamma_{eD}(G) - 1$. S' is an edge detour dominating set of G' and u is an end vertex of G’ supported by $v \in G$ implies S” is an edge detour dominating set of G. Hence, $\gamma_{eD}(G) \leq |S''| \leq \gamma_{eD}(G) - 1$, which is a contradiction. Therefore, $\gamma_{eD}(G) \leq \gamma_{eD}(G')$. Hence, $\gamma_{eD}(G) \leq \gamma_{eD}(G') \leq \gamma_{eD}(G) + 1$.

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