On \((G, D)\) - number of Special Graphs from Fan Graph

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Abstract: A subset \(D\) of \(V(G)\) is said to be a \((G, D)\) -set of \(G\) if it is both a dominating and a geodetic set of \(G\). The minimum cardinality among all \((G, D)\) sets of \(G\) is called the \((G, D)\)-number of \(G\) and is denoted by \(y_G(G)\). In this paper, the \((G, D)\)-number of Middle graph of Path, Cycle, Fan graph, Join of two Fan graphs and Total graph of Fan graph are Calculated.

Keywords – Middle graph, Total graph.

INTRODUCTION

All graphs considered here are non-trivial, simple and undirected. The order and size of a graph \(G\) are denoted by \(p\) and \(q\) respectively. A \(u - v\) path of shortest length is called a geodesic. A subset \(D \subset V(G)\) is called a geodetic set of \(G\) if every vertex in \(V - D\) lie on a geodesic joining two vertices of \(D\). A set \(D \subset V\) is said to be a dominating set in \(G\) if every vertex in \(V - D\) is adjacent to some vertex in \(D\). The domination number of \(G\) is the minimum cardinality taken over all dominating sets in \(G\) and is denoted by \(\gamma(G)\). A subset \(D \subset V(G)\) is said to be a \((G, D)\) set of \(G\) if it is both a dominating and a geodetic set of \(G\). The minimum cardinality among all \((G, D)\) sets of \(G\) is called the \((G, D)\)-number of \(G\) and is denoted by \(y_G(G)\).

2.1 Proposition:

When \(n = 2\), \(M(P_2) = P_3\) and so

\[ y_G(M(P_2)) = y_G(P_3) = 2 = n. \]

Let \(n > 2\)

Then, \(V[M(P_n)] = \{v_1, v_2, v_3, \ldots, v_n, u_1, u_2, \ldots, u_{n-1}\}\) where \(u_i\) and \(v_i\) represent the vertices and edges of the path\( P_n \) respectively.

Now, \(M(P_n)\) is as in figure 2.1

![Figure 2.1](image)

Since every \(u_i\) is adjacent to \(u_{i+1}\) and \(u_{i-1}\), we get \(S = \{v_1, v_2, v_3, \ldots, v_n\}\) as the only \((G, D)\)-set of \(M(P_n)\) and hence it is minimum.

Therefore, \(y_G(M(P_2)) = n\).

2.2 Proposition:

For \(n \geq 3\), \(y_G(M(C_n)) = n\)

Proof:

The Proof follows the same lines of Proposition 2.1

2.3 Illustration: \(y_G(M(C_6)) = 6\)
Here, \( S = \{v_1, v_2, v_3, v_4, v_5, v_6\} \) forms a minimum \((G, D)\)-set of \( G \).

Therefore, \( \gamma_G(M(C_6)) = |S| = 6 \).

### 2.4 Proposition:

\[ \gamma_G(M(F_{1,n})) = n + 1 \]

**Proof:**

Let \( V(M(F_{1,n})) = \{v_1, v_2, v_3, v_4, u_1, u_2, u_3, \ldots, u_{n-1}, e_1, e_2, \ldots, e_n\} \) where \( V(F_{1,n}) = \{v_1, v_2, \ldots, v_n\} \).

\( e_1, e_2, \ldots, e_n \) are the vertices which represent the edges \( vv_i \) for \( i = 1,\ldots,n \) and \( u_1, u_2, u_3, \ldots, u_{n-1} \) are the vertices which represent the edges \( v_{i+1} \) for \( i = 1,2,\ldots,n-1 \) as shown in figure 2.3.

Now, in \( M(F_{1,n}) \), \( d(vv_i) = 2 \).

Also, every \( e_1, e_2, \ldots, e_n \) lie on some \( uv_i \) - geodesic.

Further, for \( i = 1 \) to \( n \) the \( v_i v_{i+1} \) geodesics cover the vertices \( u_1, u_2, u_3, \ldots, u_{n-1} \). Therefore,

\[ S = \{v_1, v_2, \ldots, v_n\} \] is a geodetic set of \( M(F_{1,n}) \).

Further, \( S \) dominates all the vertices of \( M(F_{1,n}) \).

Hence, \( S \) is a \((G, D)\) - set of \( M(F_{1,n}) \). Also, this is a minimum \((G, D)\) - set of \( M(F_{1,n}) \).

Hence \( M(F_{1,n}) = |S| = n + 1 \)

### 2.5 Proposition:

Let \( G \) be a graph isomorphic to the join of two fan graphs \( F_{1,m} \) and \( F_{1,n} \).

Then, \( \gamma_G(G) = \begin{cases} \left\lfloor \frac{m+n}{2} \right\rfloor & \text{if } m + n \text{ are odd} \\ \frac{m+n+1}{2} & \text{otherwise} \end{cases} \)

**Proof:**

Let \( V(G) = \{v_1, v_2, \ldots, v_m, u_1, u_2, u_3, \ldots, u_n\} \).

Where \( u, u_1, u_2, u_3, \ldots, u_m \) are the vertices of \( F_{1,m} \) and \( v, v_1, v_2, \ldots, v_n \) are the vertices of \( F_{1,n} \). In \( G \), \( d(uv) = 1 \) for almost all the vertices and \( d(uv) = 2 \) iff \( u \) and \( v \) are alternate vertices of the \( \{v, v_1, v_2, \ldots, v_m, u, u_1, u_2, u_3, \ldots, u_n\} \)

**Case 1:**

\( m + n \) is even

Subcase 1a: \( m \) is odd and \( n \) is even

Here, \( S = \{u, u_1, u_3, \ldots, u_m, v_2, v_4, \ldots, v_{n-1}\} \) is the minimum \((G, D)\) - set of \( G \) and so \( \gamma_G(G) = |S| = \left\lfloor \frac{m}{2} \right\rfloor + \frac{n}{2} \).

Subcase 1b: \( m \) is even and \( n \) is odd

Here, \( S = \{u_1, u_3, u_5, \ldots, u_{m-1}, v_1, v_3, v_5, \ldots, v_n\} \) is a minimum \((G, D)\) - set of \( G \) and so
\[ \gamma_G(G) = \frac{m}{2} + \left\lfloor \frac{n}{2} \right\rfloor = \frac{m+n}{2} \].

**Case 2:** m+n is even

Subcase 2a: m \& n are odd.

Here, \( S = \{u_1, u_3, \ldots, u_m, v_2, v_4, \ldots, v_{n-1}, v_n\} \) is the minimum \((G,D)\) - set of G and so

\[ \gamma_G(G) = |S| = \frac{m}{2} + \frac{n+1}{2} \]
\[ = \frac{m+n+1}{2}. \]

Subcase 2b: m\&n are even.

Here, \( S = \{u_1, u_3, \ldots, u_{m-1}, v_1, v_3, \ldots, v_{n-1}, v_n\} \) is the minimum \((G,D)\) - set of G and so

\[ \gamma_G(G) = |S| = \frac{m}{2} + \left\lfloor \frac{n}{2} \right\rfloor \]
\[ = \frac{m+n+1}{2}. \]

Therefore, \( \gamma_G(G) = |S| = 3 \).

![Figure 2.5](image_url)

**Figure 2.5**

**2.6 Preposition:**

\( \gamma_G(T(F_{1,3})) = 3 \), where \( T(F_{1,3}) \) represents the Total graph of the Fan \( F_{1,3} \).

**Proof:**

\( T(F_{1,3}) \) is as in figure 2.6

Here, \( S = \{v_1, v_4, e_2, e_3, u_2\} \) forms a minimum \((G,D)\)-set of G.

Therefore, \( \gamma_G(G) = |S| = 5 \).

![Figure 2.6](image_url)

**Figure 2.6**

**2.7 Preposition:**

\( \gamma_G(T(F_{1,4})) = 5 \), where \( T(F_{1,4}) \) represents the Total graph of the Fan \( F_{1,4} \).

**Proof:**

\( T(F_{1,4}) \) is as in figure 2.6

Here, \( S = \{v_1, v_4, e_2, e_3, u_2\} \) forms a minimum \((G,D)\)-set of G.

Therefore, \( \gamma_G(G) = |S| = 5 \).

![Figure 2.7](image_url)

**Figure 2.7**

**CONCLUSION:**

We continue our work on the generalization for the total graph of \( F_{1,n} \).

**REFERENCE:**


