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On *(G, D)* - number of Special Graphs from Fan Graph

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***Abstract:*--** **A subset D of $V(G)$ is said to be a (G, D) –set of G if it is both a dominating and a geodetic set of G. The minimum cardinality among all (G, D) sets of G is called the (G, D)-number of G and is denoted by $\gamma_G(G)$. In this paper, the (G, D)- number of Middle graph of Path, Cycle, Fan graph, Join of two Fan graphs and Total graph of Fan graph are Calculated.**

Keywords – Middle graph, Total graph.

INTRODUCTION

All graphs considered here are non-trivial, simple and undirected. The order and size of a graph G are denoted by p and q respectively. A $u - v$ path of shortest length is called a geodesic. A subset $D \subset V(G)$ is called a geodetic set of G if every vertex in $V - D$ lie on a geodesic joining two vertices of D. A set $D \subset V$ is said to be a dominating set in G if every vertex in $V - D$ is adjacent to some vertex in D. The domination number of G is the minimum cardinality taken over all dominating sets in G and is denoted by $\Upsilon(G)$. A subset D of $V(G)$ is said to be a (G, D) set of G if it is both a dominating and a geodetic set of G. The minimum cardinality among all (G,D) set of G is called the (G,D)-number of G and is denoted by $\gamma_G(G)$[3]. The join of two graphs G_1 and G_2 is the graph G = G_1+G_2 with vertex set V= $V_1 \cup V_2$ and E=$E_1 \cup E_2 \cup \{uv: u \in E_1, v \in E_2\}$. A Fan graph $F_{m,n}$ is defined as the graph join $\overline{k_m} + P_n$ where $\overline{k_m}$ is the empty graph on m vertices and P_n is the path graphs on n vertices[4] and P_n+2K_1 is called the Double Fan graph. The Middle graph [1] of G is defined with the vertex set $V(G) \cup E(G)$ where two vertices are adjacent iff they are either adjacent edges of G or one is the vertex and the other is an edge incident with it and it is denoted by $M(G)$. The Total graph [1] of G has vertex set $V(G) \cup E(G)$ and edges joining all elements of this vertex set which are adjacent or incident in G. For basic definitions and terminologies we refer [2].

2.1 Preposition:

For n ≥ 2, $\gamma_G(M(P_n))$ = n

Proof:

When n = 2 , M(P_2) = P_3 and so

$\gamma_G(M(P_2)) = \gamma_G(P_3) = 2 = n.$

Let n > 2

Then, $V[M(P_n)] = \{v_1, v_2, v_3, \dots . v_n, u_1, u_2, \dots u_{n-1}\}$ where u_i and v_i represent the vertices and edges of the path P_n respectively.

Now, $M(P_n)$ is as in figure 2.1

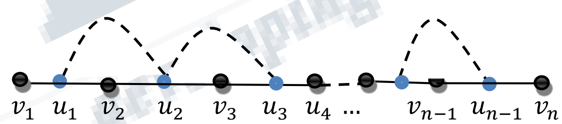

Figure 2.1

Since every u_i is adjacent to u_{i+1} and u_{i-1} , we get $S = \{v_1, v_2, v_3, \dots . v_n\}$ as the only (G, D)-set of $M(P_n)$ and hence it is minimum.

Therefore, $\gamma_G(M(P_2)) = n$.

2.2 Preposition :

For n ≥ 3, $\gamma_G(M(C_n)) = n$

Proof :

The Proof follows the same lines of Preposition 2.1

2.3 Illustration: $\gamma_G(M(C_6)) = 6$

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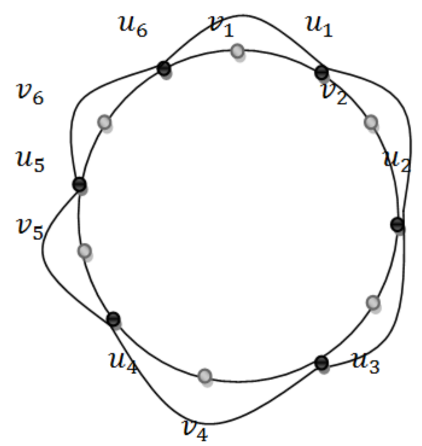

Figure 2.3

Here, $S = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ forms a minimum (G, D)-set of G.
Therefore, $\gamma_G(M(C_6)) = |S| = 6$.

2.4 Preposition :

$$\gamma_G(M(F_{1,n})) = n + 1$$

Proof :

Let $V(M(F_{1,n})) = \{v, v_1, v_2, \ldots v_n, u_1, u_2, u_3, \ldots u_{n-1}, e_1, e_2, \ldots e_n\}$ where $V(F_{1,n}) = \{v, v_1, v_2, \ldots v_n\}$, $e_1, e_2, \ldots e_n$ are the vertices which represent the edges vv_i for i=1,…n and $u_1, u_2, u_3, \ldots . u_{n-1}$ are the vertices which reperesent the edges $v_i\, v_{i+1}$ for i=1,2,…n-1 as shown in figure 2.3.

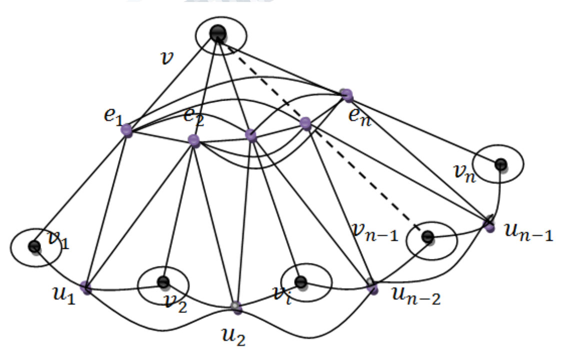

Figure 2.4

Now, in $M(F_{1,n})$, $d(vv_i) = 2$.

Also, every $e_1, e_2, \ldots e_n$ lie on some uv_1 –geodesic.

Further, for i=1 to n the $v_i v_{i+1}$ geodesics cover the vertices $u_1, u_2, u_3, \ldots . u_{n-1}$. Therefore,

$S = \{v, v_1, v_2, \ldots v_n\}$ is a geodetic set of $M(F_{1,n})$. Further, S dominates all the vertices of $M(F_{1,n})$.

Hence, S is a (G, D) - set of $M(F_{1,n})$. Also, this is a minimum (G, D) - set of $M(F_{1,n})$

Hence $M(F_{1,n}) = |S| = n + 1$

2.5 Preposition :

Let G be a graph isomorphic to the join of two fan graphs $F_{1,m}$ and $F_{1,n}$.

Then, $\gamma_G(G) = \begin{Bmatrix} \left\lceil \frac{m+n}{2} \right\rceil \; if\ m+n\ are\ odd \\ \left\lceil \frac{m+n+1}{2} \right\rceil \quad \text{otherwise}hh \end{Bmatrix}$

Proof:

Let $V(G) = \{v, v_1, v_2, \ldots v_n, u, u_1, u_2, u_3, \ldots u_m\}$

Where $u, u_1, u_2, u_3, \ldots u_m$ are the vertices of $F_{1,m}$ and $v, v_1, v_2, \ldots v_n$ are the vertices of $F_{1,n}$. In G, $d(uv) = 1$ for almost all the vertices and $d(uv) = 2$ iff $u\ and\ v$ are alternate vertices of the $\{v, v_1, v_2, \ldots v_n, u, u_1, u_2, u_3, \ldots u_m\}$

Case 1:

$$m + n\ is\ odd$$

Subcase 1a :

m is odd & n is even

Here,
$s = \{u, u_1, u_3, \ldots u_m, v_2, v_4, \ldots v_n\}$ is the minimum (G, D)- set of G and so $\gamma_G(G) = |S| = \left\lceil \frac{m}{2} \right\rceil + \frac{n}{2}$
$= \left\lceil \frac{m+n}{2} \right\rceil$.

Subcase 1b:

m is even & n is odd

Here, $S = \{u_1, u_3, u_5, \ldots u_{m-1}, v_1, v_3, v_5, \ldots v_n\}$ is a minimum (G, D)- set of G and so

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$\gamma_G(G) = \frac{m}{2} + \left\lceil\frac{n}{2}\right\rceil = \left\lceil\frac{m+n}{2}\right\rceil$.

Case 2:

m+n is even

Subcase 2a:

m & n are odd.

Here, $S = \{u_1, u_3, \dots u_m, v_2, v_4, \dots v_{n-1}, v_n\}$ is the minimum (G, D) - set of G and so

$\gamma_G(G) = |S| = \left\lceil\frac{m}{2}\right\rceil + \frac{n+1}{2}$

$= \left\lceil\frac{m+n+1}{2}\right\rceil$.

Subcase 2b:

m&n are even.

Here,

$S = \{u_1, u_3, \dots u_{m-1}, v_1, v_3, \dots v_{n-1}, v_n\}$is the minimum (G, D) - set of G and so

$\gamma_G(G) = |S| = \frac{m}{2} + \left\lceil\frac{n+1}{2}\right\rceil$

$= \left\lceil\frac{m+n+1}{2}\right\rceil$.

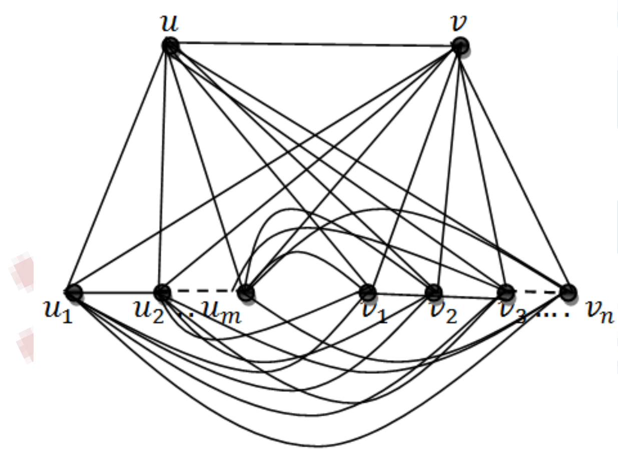

Figure 2.5

2.6 Preposition :

$\gamma_G(T(F_{1,3})) = 3$, where $T(F_{1,3})$ represents the Total graph of the Fan $F_{1,3}$.

Proof:

$T(F_{1,3})$ is as in figure 2.6

Here,

$S = \{v_1, v_3, e_1\}$ forms a minimum (G, D)-set of G.

Therefore, $\gamma_G(G) = |S| = 3$.

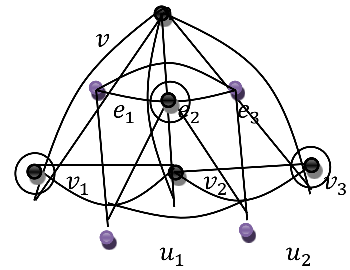

Figure 2.6

2.7 Preposition :

$\gamma_G(T(F_{1,4})) = 5$, where $T(F_{1,4})$ represents the Total graph of the Fan $F_{1,4}$.

Proof:

$T(F_{1,4})$ is as in figure 2.6

Here,

$S = \{v_1, v_4, e_2, e_3, u_2\}$ forms a minimum (G, D)-set of G.

Therefore, $\gamma_G(G) = |S| = 5$.

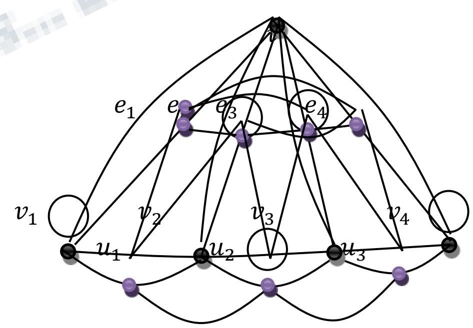

Figure 2.7

CONCLUSION :

We continue our work on the generalization for the total graph of $F_{1,n}$

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