ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 2, Issue 12, December 2017

On Total Detour Number Of Some Path related Graphs

[1] M.Selva Lakshmi, [2] K. Palani
[1] M.Phil Scholar, [2] Associate Professor of Mathematics
[1][2] A.P.C Mahalaxmi College for Women, Thoothukudi, Tamilnadu, India.

Abstract:-- **For a connected graph G = (V,E), a detour set S ⊆ V(G) is called a total detour set of G if the subgraph G[S] induced by S has no isolated vertices. The total detour number tdn(G) of G is the minimum order of its total detour sets and any total detour set of order tdn(G) is called a td-set of G. In this paper, the total detour number of some path related graphs are found and compared with their detour numbers.**

Keywords – Detour number and Total detour number

INTRODUCTION

By a graph G = (V,E), we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Throughout this paper, we consider only connected graphs with at least two vertices. For basic definitions and terminologies we refer [2].

For vertices u and v in a connected graph G, the detour distance D(u,v) is the length of the longest u-v path in G. A u-v path of length D(u,v) is called a u-v detour.

A vertex x is said to lie on a u-v detour P if x is a vertex of a u-v detour path P including the vertices u and v. A set S ⊆ V is called a detour set if every vertex v in G lie on a detour joining a pair of vertices of S. Let G = (V,E) be a connected graph with at least two vertices. A set S ⊆ V(G) is called a total detour set of G if S is a detour set of G and the subgraph G[S] has no isolated vertices. The total detour number tdn(G) of G is the minimum order of its total detour set. A total detour set of order tdn(G) is called a td-set of G[3]. Let $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ be any two graphs. The join of G_1 and G_2 is the graph $G = G_1 + G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2 \cup \{uv : u \in V_1, v \in V_2\}$. The graph $P_n + K_1$ is called a Fan. $K_{1,m}$:n is the graph obtained from $K_{1,m}$ by replacing the 'm' pendant edges by m paths of length n[4]. The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2. The graph $P_n \odot K_1$ is called a Comb.Sp(P_n,$K_{1,m}$) is a graph in which the root of the star $K_{1,m}$ is attached at one end of the path P_n[1]. Z-P_n is the graph obtained from, two paths P_n and P_n' of same length 'n', by joining the i^{th} vertex of P_n to the i-1^{th} vertex of P_n'. The resulting graph is denoted as Z-P_n.

The following theorems used in the sequel are from [3].

Theorem 1.1 For a connected graph G of order p≥ 2, 2 ≤ dn(G) ≤ tdn(G) ≤ p for some vertex x in G.

Theorem 1.2 Any total detour set contains all the end vertices of G.

2. Total Detour Number Of Some Path related Graphs:

Proposition 2.1: tdn(P_n+K_1) = 2.

Proof: Let V(P_n+K_1)={$u,v_1,v_2,v_3,v_4,v_5,\ldots,v_n$} where u is the vertex of K_1. Then, P_n+K_1 is as in Figure 2.1.

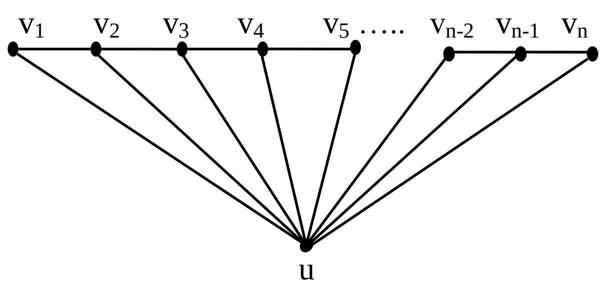

Figure 2.1 (P_n+K_1)

ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 2, Issue 12, December 2017

Let G= P_n+K_1. Clearly, S = {u,v_n} is a total detour set of P_n+K_1 since every other vertex of P_n+K_1 lie on a detour joining u and v_n & G[S] has no isolated vertices. Further, by theorem 1.1, S is a minimum total detour set of P_n+K_1. Therefore, tdn(P_n+K_1) = |S| = 2

Proposition 2.2: tdn($P_n \odot K_1$) = 2n

Proof: LetV($P_n \odot K1$)={$u_1,u_2,u_3,..,u_n,v_1,v_2,v_3,..,v_n$} where $v_1,v_2,v_3,\ldots,v_n$ are the vertices of path. Then, $P_n \odot K_1$ is as in Figure 2.2.

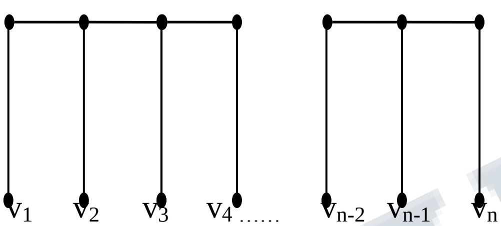

Figure 2.2 ($P_n \odot K_1$)

Let G = $P_n \odot K_1$, for all n ≥ 2. By theorem 1.2, S = {$v_1,v_2,v_3,v_4,\ldots..,v_{n-1},v_n$} is contained in any total detour set of G. But, S is not a total detour set since all its vertices are isolated in G[S]. Also, S is a detour set of G. Therefore, to get a total detour set of G from S, support vertex of each vertex in S is to be included. Therefore, D={$u_1,u_2,u_3,u_4,\ldots,u_{n1},u_n,v_1,v_2,v_3, v_4,\ldots..,v_n$} is a total detour set and further it is minimum. Therefore, tdn(G) = 2n.

Proposition 2.3 tdn(Sp($P_n,K_{1,m}$) = m+3

Proof: Let V(Sp($P_n,K_{1,m}$)={$u_1,u_2,u_3,..,u_n, v_1,v_2,v_3,\ldots,v_{m-1},v_m$} where $v_1,v_2,v_3,\ldots,v_{m-1},v_m$ are the pendent vertices of $K_{1,m}$. Then, Sp($P_n,K_{1,m}$) is as in Figure 2.3.

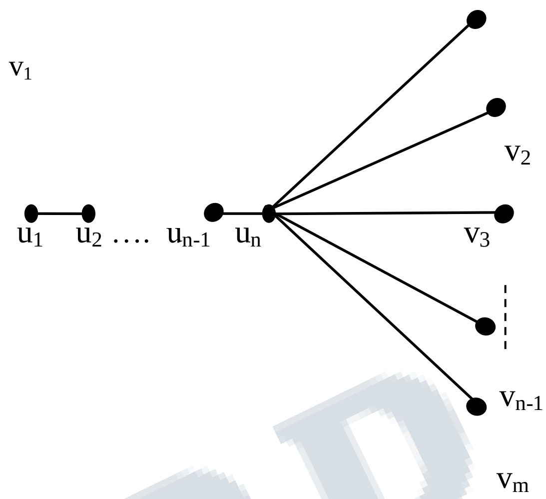

Figure 2.3 Sp($P_n,K_{1,m}$)

Let G = Sp($P_n,K_{1,m}$),for all n ≥ 2. Proceeding as in proposition2.2, since u_n is the support vertex of all v_i and u_2 support u_1, tdn(Sp($P_n,K_{1,m}$)= m+3.

Proposition 2.4 tdn($K_{1,m}$: n) = 2m

Proof:

LetV($K_{1,m}$:n)={$u,v_{11},v_{12},..,v_{1n},v_{21},v_{22},..,v_{mn}$} where u is the vertex of $K_{1,m}$. Then, $K_{1,m}$: n is as in Figure 2.4.

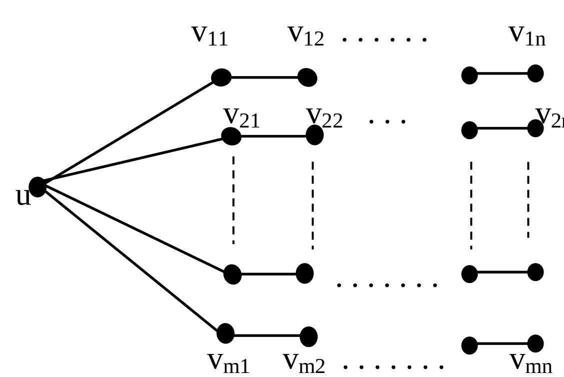

Figure 2.4 ($K_{1,m}$: n)

Let $G=(K_{1,m} : n)$, for all $n\geq 2$. Since the set of end vertices $\{v_1,v_2,..,v_{mn}\}$ is a detour set, proceeding as in proposition 2.2, $tdn(K_{1,m} : n) = 2n$.

Proposition 2.5 $tdn(P_n \odot K_2) = 2n$

Proof: Let $V(P_n \odot K_2)=\{v_1,v_2,..,v_n,u_1,u_2,..,u_{2n}\}$ where $u_1,u_2,..,u_{2n}$ are the vertices of n copies of K_2. Then, $P_n \odot K_2$ is as in Figure 2.5.

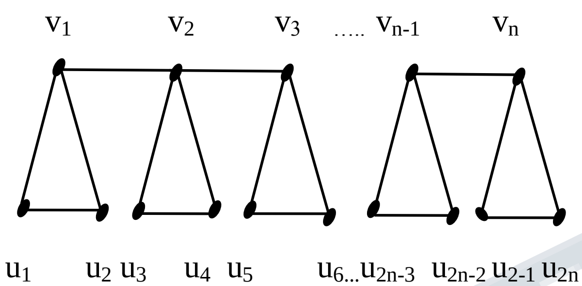

Figure 2.5$(P_n \odot K_2)$

Let $G= P_n \odot K_2$. Clearly, $S=\{u_1,u_3,u_5,\ldots, u_{2n-1}\}$ forms a minimum detour set of $P_n \odot K_2$. Also, all the vertices in S are isolated in G[S]. Therefore, S is not a total detour set. Further, the graph contains no vertex adjacent to at least two vertices of S. Therefore, to make S as a total detour set we need a minimum of |S| vertices. Therefore, $S\cup\{u_2,u_4,u_6,\ldots,u_{2n}\}$ & $S\cup\{v_1,v_2,\ldots,v_n\}$ forms a minimum total detour set of $P_n \odot K_2$. Hence $tdn(P_n \odot K_2) = |S| + n = 2n$.

Theorem 2.6 $tdn(P_n \odot K_m) = 2n$ for every $m \geq 3$.

Proof: Clearly, any set S containing at least one vertex from each K_m forms a minimum detour set and each vertex of S is isolated in G[S]. Also, there is no vertex in V-S adjacent to at least two vertices of S.

Therefore, $tdn(P_n \odot K_m) \geq |S| + |S|$

$= 2n$ (Since there are n 'K_m' in $P_n \odot K_m$) (1)

Also, it is clear that any set with $\geq 2n$ vertices forms a total detour set of $P_n \odot K_m$.

Therefore, $tdn(P_n \odot K_m) \leq 2n$ (2)

By (1) and (2), $tdn(P_n \odot K_m) = 2n$.

Proposition 2.7 $tdn(Z\text{-}P_n) = 4$

Proof: Let $V(Z\text{-}P_n)=\{v_1,v_2,..,v_n,v_1',v_2',..,v_n'\}$ Then, $Z\text{-}P_n$ is as in Figure 2.7

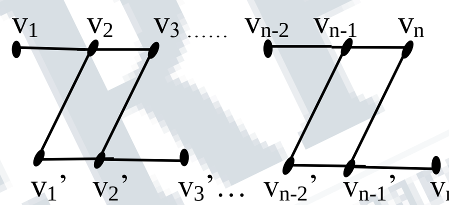

Figure 2.7 $(Z\text{-}P_n)$

For $Z\text{-}P_n$, $S = \{v_1,v_n'\}$ is a detour set. But, it is not a total detour set since v_1 and v_n' are isolated vertices in $G[v_1,v_n']$. Further, if any one of the other vertices is included in S, even then S contains at least one isolated vertex. But, if the vertices v_2 and v_2' are added to S, then S contains no isolated vertex and hence S become a total detour set. Obviously, it is a minimum total detour set of $Z\text{-}P_n$. Therefore, $tdn(Z\text{-}P_n) = |S| + 2 = 4$.

REFERENCES:

[1] A.Nellai Murugan, Latha Martin and S.Heera John, Path related 1-Near mean square cordial graphs, Enrich, VIII(I):45-52, July – December, 2016.

[2] F.Harary, Graph Theory, Addison-Wesley, 1969.

ISSN (Online) 2456 -1304

International Journal of Science, Engineering and Management (IJSEM)
Vol 2, Issue 12, December 2017

[3] J.John and N.Arianayagam, Total Detour Number Of A Graph, Journal Of Discrete Mathematical Sciences And Cryptography, Vol.17(2014), No.4, PP.337-350.

[4] Palani.K and Nagarajan.A, (G, D)-Number of Graph, International Journal of Mathematics Research. ISSN 0976-5840 Volume 3(2011), pp 285-299.

