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Indegree Prime Labeling of Digraphs

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Abstract:-- **Let D(p,q)be a digraph. A functionf : V→{1,2,…,p+q} is said to be an indegree prime labeling of D if at each $v \in V(D)$, gcd[$f(u), f(v)$] = 1 , ∀ $\overrightarrow{uv} \in E(D)$.In this paper, it is found that whether the digraphs nP$_2$,nP$_4$,$P_m \cup P_n \cup P_r$, directed snake TS_n and directed cycle C_n admit indegree prime labeling or not. Further, the same is tried for full directed binary treeT_{31}.**

Keywords – Indegree prime labeling

1. INTRODUCTION

Throughout this paper, we consider only finite and directed graphs. A labeling of a graph G is an assignment of integers to either the vertices or the edges or both subject to certain conditions.A directed graph or digraph D consists of a finite set V of vertices (points) and a collection of ordered pairs of distinct vertices. Any such pair (u, v) is called an arc or directed line and will usually be denoted by $\overrightarrow{uv}$. The arc $\overrightarrow{uv}$ goes from u to v and incident with u and v, we also say u is adjacent to v and v is adjacent from u. A digraph D with p vertices and q arcs is denoted by D(p, q).A directed cycle is a nontrivial closed walk with all its vertices distinct. Adirected triangular snake TS_n is a connected digraph, all of whose blocks are directed triangles and whose block-cut points form a directed path. A directed tree is connected digraph without cycles. The indegree $d^-(v)$ of a vertex v in a digraph D is the number of arcs having v as its terminal vertex. The outdegree $d^+(v)$ of v is the number of arcs having v as its initial vertex [4].. A directed tree in which one vertex(root) is distinguished from all the others is called a rooted tree. A rooted tree with no arcs directed towards the root is called out-tree. An out-tree is said to be directed full binary tree if the outdegreeof every vertex is exactly two except for the pendant vertices and theoutdegree is zero for pendant vertices.X(a,b) represents the digraph D(V,E) on 2a+b vertices with a≥1& b≥1,V(D) = $X \cup Y \cup Z$, where $X = \{x_1, x_2, \dots, x_a\}$;$Y = \{y_1, y_2, \dots, y_a\}$$Z = \{z_1, z_2, \dots, z_b\}$ and $E\ (D) = \{\overrightarrow{x_j y_j}\ , \overrightarrow{y_j x_j} \big| j = 1,2, \dots, a\}\ \cup \{\overrightarrow{x_j z_l}, \overrightarrow{z_l y_j}\ | j = 1,2, \dots, a\ \&\ l = 1,2, \dots, b\ \}$[3].

Let G = G(V,E) be a graph. A bijection f: V→{1,2,3,…..,|V|} is called prime labelling if for each e={u,v} belong to E, we have gcd[f(u),f(v)]=1 [5].
Motivated by this definition we attempt to define the same concept for digraphs. As a first step, we define the new concept indegree prime labeling for digraphs. This paper includes the definition and proof of "The directed cycle C_n (n ≥ 3), directed triangular snake $TS_{n,}$, directed paths nP$_2$,nP$_4$,Pm $\cup P_n \cup P_r$ exhibits indegree prime labeling". Also, it is proved that full directed binary tree T_{31} admitindegree prime labeling. In our further work we planned to define outdegree prime labeling in a similar way and then to move to prime labeling. For more reference on labeling, we refer [1].

Remark 1.1 gcd of any two consecutive natural number is 1. [2]

Remark 1.2 gcd of any two consecutive odd natural number is 1. [6]

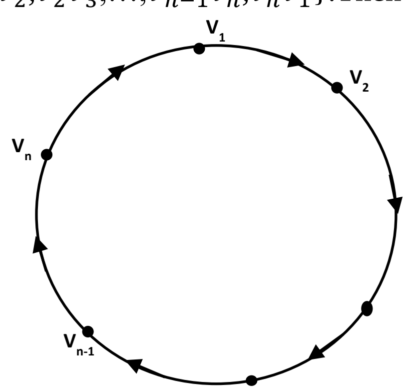
2. MAIN RESULTS :

Indegree prime labeling

Definition2.1 Let D(p,q) be a digraph. A function f:V→{1,2,…,p+q} is said to bean indegree prime labeling of D if at each $v \in V(D)$,gcd[$f(u), f(v)$] = 1 , ∀ $\overrightarrow{uv}$

Theorem 2.2 Directed cycle C_n(n ≥ 3) admits indegree prime labeling

Proof Let C_n be a directed cycle oflength n with vertexset V= $\{v_1, v_2, \dots, v_{n-1}, v_n\}$and the arc set be E($C_n$) =$\{\overrightarrow{v_1 v_2}, \overrightarrow{v_2 v_3}, \dots, \overrightarrow{v_{n-1} v_n}, \overrightarrow{v_n v_1}\}$.Then the directed



cycle C_n is as in figure 2.1

Figure 2.1(Directed Cycle C_n)

Here, p+q=2n

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Define a functionf: V→ {1, 2,…, 2n} byf (v_i)= i , 1 ≤ i ≤ n

For every i,indegree of v_i is 1, since $\overrightarrow{v_{i-1}v_i}$ is the only arc giving indegree to v_i.

Then, by remark 1.1,

∀i, gcd [f(v_i) , f(v_{i+1})] = gcd [i , i+1] = 1

Therefore, the map f(v_i)=i gives an indegree prime labeling of any cycle C_n.

Hence, Any cycle C_n (n ≥ 3) admits an indegree prime labeling.

Theorem 2.3 The directed triangular snake TS_n admits indegree prime labeling

Proof Let V(TS_n) ={v_1,v_2,…,v_{2n-2},v_{2n-1}} andE(TS_n)={$\overrightarrow{v_iv_{i+1}}$ | 1 ≤ i ≤ 2n-2}∪ {$\overrightarrow{v_iv_{i+2}}$ | i = 1,3,5,…,2n-3}, where vertices with odd suffixes lie along the path and the vertices with even suffixes are on the top of the triangles.Then the directed triangular snake is as in the figure 2.1.

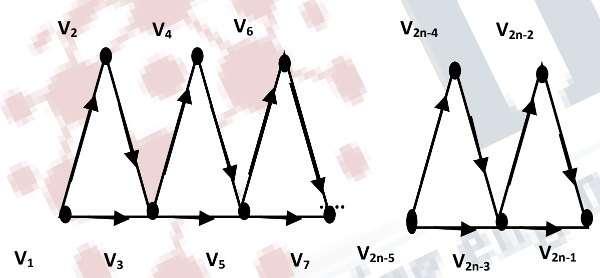

Figure 2.2

(Directed Triangular Snake TS_n)

Hence, the directed triangular snake has 2n-1 vertices and 3n-3 arcs.

Define a function f : V→{1,2,3,…,5n-4} by$f(v_i) = i$,∀1 ≤ i ≤ $2n - 1$.

i.e., The vertices on the horizontal path get the consecutive odd numbers and the upper vertices in the triangles get the consecutive even numbers.

Here, $d^-(v_1) = 0$ and
$d^-(v_{2i}) = 1$, ∀ $i = 1,2,\dots,n-1$

Further, the arc corresponding to $d^-(v_{2i})$is $\overrightarrow{v_{2i-1}v_{2i}}$.
Therefore, by remark1.1,
gcd [$f(v_{2i-1}), f(v_{2i})$]= gcd [$2i - 1$, $2i$]=1 ...(1)

Also, $d^-(v_{2i+1}) = 2$, $\forall i = 1,2,\dots,n-1$ and the corresponding arcs are $\overrightarrow{v_{2i}v_{2i+1}}$ and $\overrightarrow{v_{2i-1}v_{2i+1}}$

Therefore, the corresponding function values are either two consecutive integers or two consecutive odd integers.
Therefore,byremark1.1&1.2, ∀ $v = v_{2i+1}$ *and* $\overrightarrow{uv} \in E(TS_n)$,

gcd[$f(u), f(v)$]=1 …(2)

From(1)and(2), ∀$v \in v(TS_n)$,

$$\gcd[f(u), f(v)] = 1, \forall \overrightarrow{uv} \in E(TS_n)$$

Therefore, the map $f(v_i) = i$,
∀ 1 ≤ i ≤ $2n - 1$gives an indegree prime labeling of any directed triangular snake TS_n.

Hence, Any directed triangular snake TS_nadmits indegree prime labeling

Theorem 2.4 Directed path nP_2 admits indegree prime labeling

Proof Let $V = \{u_i, v_i | 1 \le i \le n\}$ be the vertex set and $E(nP_2) = \{\overrightarrow{u_iv_i} | 1 \le i \le n\}$ be the arc set of the directed path nP_2.Then the directed path nP_2 is as in the figure 2.3.

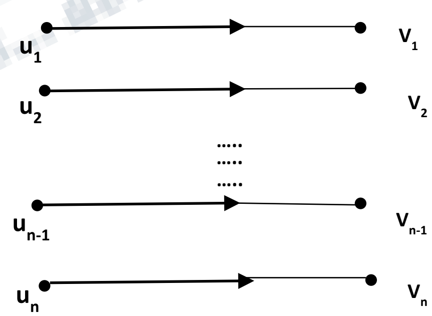

Figure 2.3(Directed Path nP_2)

Therefore, p+q=3n
Define a function$f : V \to \{1,2,3,\dots,3n\}$ by

$f(u_i) = 2i - 1$ and$f(v_i) = 2i$,
∀ 1
Here, $d^-(u_i) = 0$, ∀ 1 ≤ i ≤ n
Further, For every i, $d^-(v_i) = 1$ and the corresponding arc is $\overrightarrow{u_iv_i}$

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Also, the corresponding function values are two consecutive integers, by remark 1.1,

gcd[$f(u_i)$, $f(v_i)$]

=gcd [$2i-1, 2i$] = 1 , $\forall\, 1 \le i \le n$

Therefore, f induces an indegree prime labeling of directed path nP_2.

Hence, Any directed path nP_2 admits an indegree prime labeling.

Theorem 2.5 Directed Path nP_4 admits indegree prime labeling

Proof Let $V = \{u_i, v_i, w_i, x_i \mid 1 \le i \le n\}$ be the vertex set and E(nP4) $= \{\overrightarrow{u_i v_i}, \overrightarrow{v_i w_i}, \overrightarrow{w_i x_i} \mid 1 \le i \le n\}$ be the arc set of the directed path nP_4. Then the directed path nP_4 is as in the figure2.4

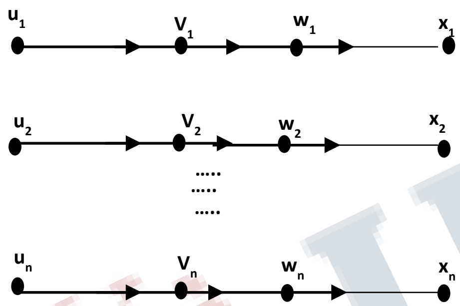

Figure 2.4 (Directed Path nP_4)

Therefore, p+q=7n

Define a function $f : V \to \{1,2,\dots,7n\}$ by $f(u_i) = 4(i-1)+1, f(v_i) = 4(i-1)+2$, $f(w_i) = 4(i-1)+3, f(x_i) = 4(i-1)+4$

$\forall\, 1 \le i \le n.$

Here, $d^-(u_i) = 0, \forall\, 1 \le i \le n$

Further , for every i, $d^-(v_i) = 1, d^-(w_i) = 1$,

$d^-(x_i) = 1$ and the corresponding arcs are $\overrightarrow{u_i v_i}, \overrightarrow{v_i w_i}, \overrightarrow{w_i x_i}$

Also, the corresponding function values are two consecutive integers,

By Remark 1.1, For every i = 1 ≤ i ≤ n,

gcd[f$(u_i), f(v_i)$]=

gcd [$4(i-1)+1, 4(i+1)+2$] = 1

gcd[$f(v_i), f(w_i)$]=

gcd[$4(i-1)+2$, $4(i-1)+3$] = 1

gcd[$f(w_i), f(x_i)$]=

gcd[$4(i-1)+3$, $4(i-1)+4$] = 1

Therefore, f induces an indegree prime labeling of nP_4.

Hence, Any directed path nP_4 admits indegree prime labeling.

Theorem 2.6 Directed path $P_m \cup P_n \cup P_r$ admits indegree prime labeling

Proof Let $V = \{u_i \mid 1 \le i \le m\} \cup \{v_j \mid 1 \le j \le n\} \cup \{w_k \mid 1 \le k \le r\}$ be the vertex set and $E\,(P_m \cup P_n \cup P_r) = \{\overrightarrow{u_i u_{i+1}} \mid 1 \le i \le m-1\} \cup \{\overrightarrow{v_j v_{j+1}} \mid 1 \le j \le n-1\} \cup \{\overrightarrow{w_k w_{k+1}} \mid 1 \le k \le r-1\}$ of the directed path $P_m \cup P_n \cup P_r$. Then the directed path $P_m \cup P_n \cup P_r$ is as in figure 2.5.

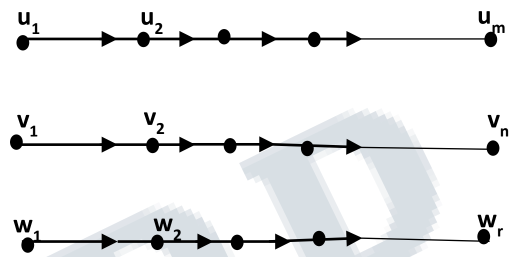

Figure 2.5

(Directed path $P_m \cup P_n \cup P_r$)

Hence, the directed path $P_m \cup P_n \cup P_r$ has m+n+r vertices and m+n+r-3 arcs.

Define a function

$f : V \to \{1,2,\dots,2(m+n+r)-3\}$ by $f(u_i) = i, \forall\, 1 \le i \le m; f(v_j) = m+j,$

$\forall\, 1 \le j \le n; f(w_k) = m+n+k,$

$\forall 1$

Here, $d^-(u_1) = d^-(v_1) = d^-(w_1) = 0$; $d^-(u_i) = 1, \forall\, 2 \le i \le m; d^-(v_i) = 1, \forall\, 2 \le i \le n$; $d^-(w_i) = 1, \forall\, 2 \le i \le r$ and the corresponding arcs are

$\overrightarrow{u_i u_{i+1}}, \forall 1 \le i \le m$, $\overrightarrow{v_i v_{i+1}}, \forall 1 \le i \le n$ & $\overrightarrow{w_i w_{i+1}}, \forall 1 \le i \le r$.

Also, the corresponding function values are two consecutive integers.Therefore, by remark 1.1, for every i = 1 to m-1, j=1 to n-1, k=1 to r-1,

$gcd[\, f(u_i), f(u_{i+1})] = \gcd[\, i, i+1] = 1$

gcd$[f(v_j), f(v_{j+1})]$

=gcd[m+j, m+j+1]=1 and

gcd[$f(w_k), f(w_{k+1})$]

= gcd[$m+n+k, m+n+k+1$]=1

Therefore, f induces an indegree prime labeling of $P_m \cup P_n \cup P_r$.

Hence, Any directed path $P_m \cup P_n \cup P_r$ admits indegree prime labeling.

Theorem 2.7 The directed full binary tree with 31 vertices (T_{31}) admits indegree prime labeling.

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Proof Given T_{31} is the full directed binary tree with 31 vertices and 30 arcs. Let V(T_{31})={v_1,v_2,…,v_{31}}. Then the full directed binary tree T_{31} is as in figure2.6

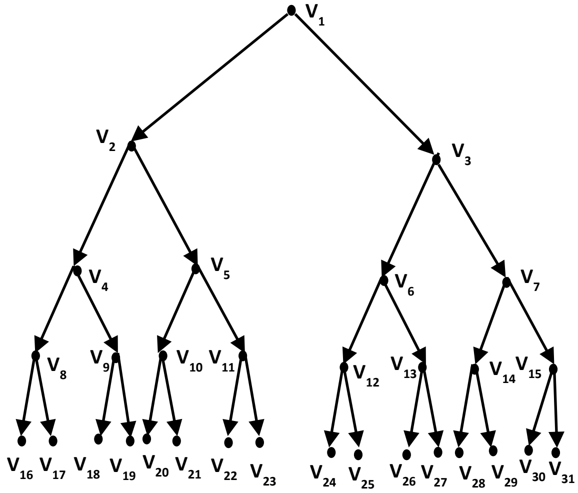

Figure 2.6

Directed full binary tree (T_{31})

Here,p+q= 30+31= 61

List out the prime numbers ≤ 61

Define f(v_i)=ith prime number ≤ 61,for all v_i except for the pendant vertices.

Now, assign the leftout non-prime numbers starting from the least to the pendant vertices.

This will leads to an indegree prime labeling of T_{31}.

Hence, T_{31} admits an indegree prime labeling.

Observation 2.9 Let T_n be a full directed binary tree with p vertices and q arcs. Suppose the number of pendant vertices in T_n is k and the number of prime numbers ≤ p+q is s. Then, T_n admits an indegree prime labeling if s ≥ n-k.

Theorem 2.10 Any digraph with p vertices and q edges with p prime numbers ≤ p+q admits indegree prime labeling.

Proof Let G=(p,q) be any digraph.

Assume that there are p primes ≤ p+q.

Since gcd [m,n]=1, For any two prime numbers m & n, assigning any set of p primes to the p vertices gives a indegree prime labeling.

Illustration 2.11

The digraph X(1,3) has 5 vertices and 8 edges. Also, there are 6 prime numbers ≤13.

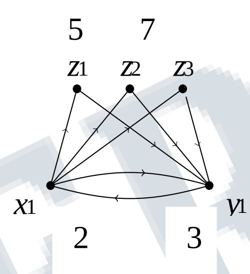

Figure 2.7

As in figure2.7, X (1,3) admits an indegree prime labeling.

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